Mean zonal flows induced by Boussinesq thermal convection in rotating spherical shells

Shin-ichi Takehiro*[1]
Youhei Sasaki[1], Ken-suke Nakajima[2], Yoshi-Yuki Hayashi[3]

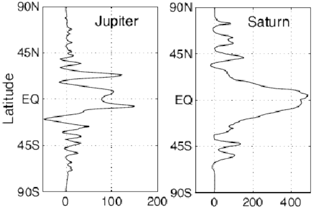
[1]Kyoto Univ. , [2]Kyushu Univ.,[3]Kobe Univ.

Outline

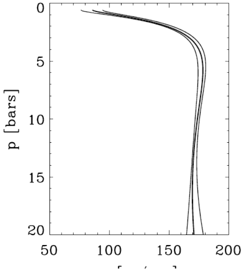
Outline

Surface flows of gas giant planets

- Surface flows of Jupiter and Saturn are characterized by the broad prograde jets around the equator and the narrow alternating jets in mid- and high-latitudes.
- It is not yet clear whether those surface jets are produced by convective motions in the “deep” region, or are the result of fluid motions in the “shallow” weather layer.



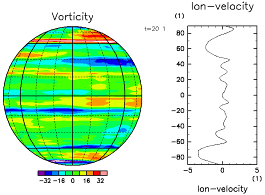
(Sukoriansky et al., 2002)

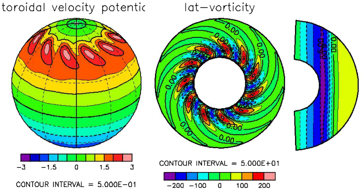


(Vasavada and Showman, 2005)

“Deep” models and “Shallow” models

- “Shallow” models
  - 2D turbulence on a rotating sphere
  - Primitive model
    - Result: Narrow alternating jets in mid- and high-latitudes.
    - Problem: the equatorial jets are not necessarily prograde
- “Deep” models
  - Convection in rotating spherical shells
    - Result: Produce equatorial prograde flows easily
    - Problem: difficult to generate alternating jets in mid- and high-latitudes





Outline

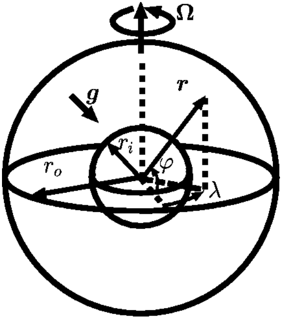
Model setup

- Boussinesq fluid in a rotating spherical shell.
  - scaling: the shell thickness, viscous diffusion time, temperature difference.

$$\nabla\cdot\boldsymbol{u} = 0,$$

$$\mathrm{E}\left\{\frac{\partial\boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla^2\boldsymbol{u}\right\} + 2\boldsymbol{k}\times\boldsymbol{u} + \nabla p = \frac{\mathrm{RaE}^2}{\mathrm{Pr}}\frac{\boldsymbol{r}}{r_o}T,$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{u}\cdot\nabla)T = \frac{1}{\mathrm{Pr}}\nabla^2 T.$$



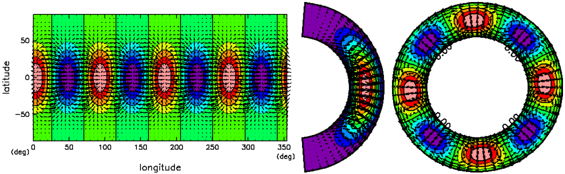
- Parameters:
  - Plandtl number: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$
  - Rayleigh number: $\mathrm{Ra} = \dfrac{\alpha g_o \Delta T D^3}{\kappa\nu}$
  - Ekman number: $\mathrm{E} = \dfrac{\nu}{\Omega D^2}$
  - radius ratio: $\eta = \dfrac{r_i}{r_o}$

Outline

Slowly rotating cases

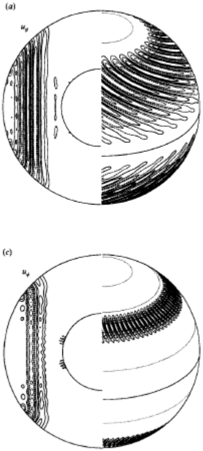
Busse (1970a)

- Asymptotic expansion with small parameter, small rotation rate
- 0th order
  - convection structure without rotation rate
  - degeneration of Y_n^m modes with the same degree n
- 1st order
  - retrograde propagation
- 2nd order
  - preference of the sectorial mode Y_n^n
  - banana-type structure



Rapidly rotating cases

- Due to the Taylor Proudman theorem, convective motion is along the rotation axis (columnar convection, Busse 1970b).
- The columns are distorted by the curvature of the outer boundary and form spiral structure.
- The tilting causes cylindrically outward Reynolds stress.
- Convection cells propagate in the prograde direction

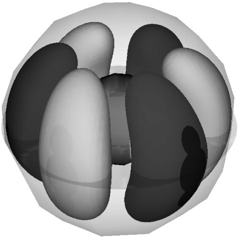


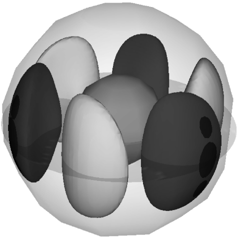
(Zhang 1992)

Banana or column

Takehiro (2010)

- Slowly rotating cases
  - Banana type structure → retrograde propagation
- Rapidly rotating cases
  - Columnar type structure → prograde propagation
- Explanation with conservation of potential vorticity





Slowly rotating case ($E_k = 10^{-1}, P_r = 1$)

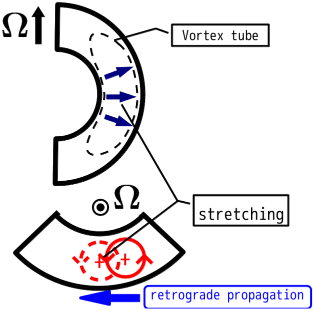
Rapidly rotating case ($E_k = 10^{-2}, P_r = 1$)

Contour surfaces of absolute value of vorticity

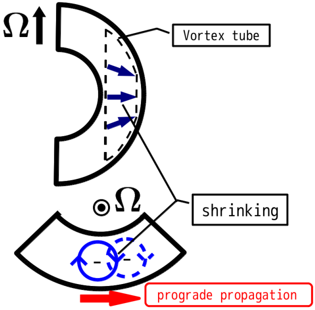
Propagation mechanism

Takehiro (2010)

- Importance of meridional structure
  - Slow (banana) : vortex tubes bend along the shell
  - Rapid (column) : vortex tubes extend along the rotation axis, suppressed by the outer spherical boundary.



Slowly rotating case



Rapidly rotating case

Spiral structure and Rossby waves

Reason why spiral structure appears?

- Dispersion relation of Rossby waves

$$\omega = -\frac{\beta k}{k^2 + l^2}$$

- Incorrect explanation
  - Tilting of the boundaries larger in the outer region

  $\rightarrow$ Topographic β large

  $\rightarrow$ Phase speed of the waves faster

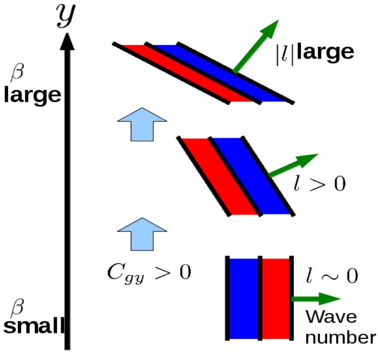
  $\rightarrow$ Tilting of the phase
- Phase speed must be identical in critical modes
- Correct explanation
  - an increase of the radial wavenumber in the outer region due to conservation of the phase speed.

Spiral structure and Rossby waves

- Variation of β in y direction $\rightarrow$ variation of y-wavenumber l (ω, k are conserved).

$$l(y) = \pm\sqrt{-k^2 - \frac{\beta(y)k}{\omega}}$$

- Large $\beta(y) \rightarrow$ Large $l(y)$ $\rightarrow$ strong tilting of the phase in the outward region $\rightarrow$ spiral structure emerges
- Direction of the tilting
  - Rossby waves propagate outward (the inner region is convective unstable)
  - Group velocity $C_{gy} = 2\beta kl/(k^2 + l^2)^2$ must be positive $\rightarrow$ positive $l(y)$



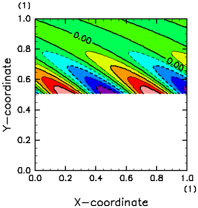
(Takehiro and Irie, 2001)

Spiral structure and Rossby waves : dependency of the Prandtl number

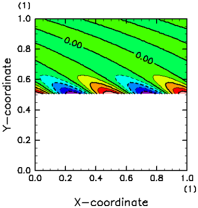
Takehiro (2008)

- Small P_r → small viscosity → longer propagation of waves → spiral structure
- Large P_r → large viscosity → waves dissipate quickly, shorter propagation → columnar structure
- Calculation with WKB theory

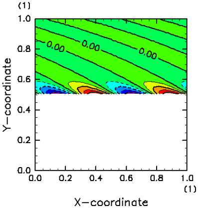
$$\psi = \psi_0 e^{i[kx + \int l(y)dy - \omega t]} \cdot e^{-\int P_r (k^2 + l(y)^2)/C_{gy}(y)dy}$$



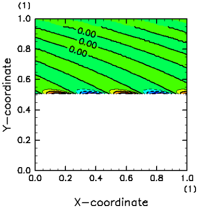
$P_r = 0.1$



$P_r = 0.3$



$P_r = 0.5$



$P_r = 1.0$

Outline

Weak non-linear analysis

- Obtaining linear responses to the non-linear terms into which are substituted the critical solution

  (cf. Takehiro and Hayashi 1999)

  - Original equations :

$$\frac{\partial \boldsymbol{x}}{\partial t} + L\boldsymbol{x} + N[\boldsymbol{x}] = 0,$$

  - Critical solution (linearized equations) :

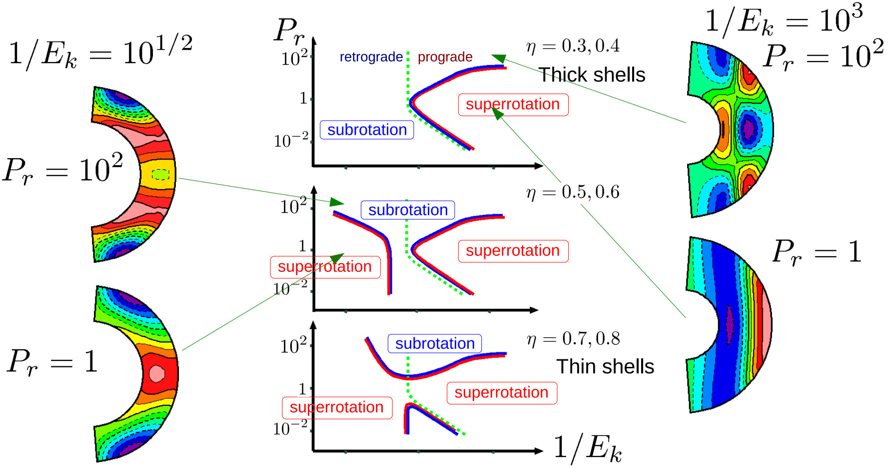
$$\frac{\partial \boldsymbol{x}^{(1)}}{\partial t} + L\boldsymbol{x}^{(1)} = 0,$$

  - Weak non-linear fields :

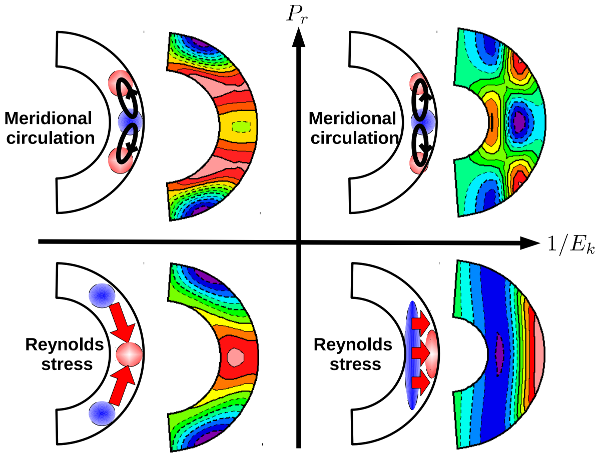
$$\overline{L\boldsymbol{x}^{(2)}} = -\overline{N[\boldsymbol{x}^{(1)}]}.$$

Generation of mean zonal flow
~ weak nonlinear calculations

Takehiro and Hayashi (1999)



Generation of mean zonal flow
$\sim$ weak nonlinear calculations



- Large P_r : meridional circulation
  $\Downarrow$
  equatorial sub-rotation
- Small P_r : Reynolds stress
  $\Downarrow$
  equatorial super-rotation

Slowly rotating cases with large P_r

AM transport by mean meridional circulations

- Convective heat transport

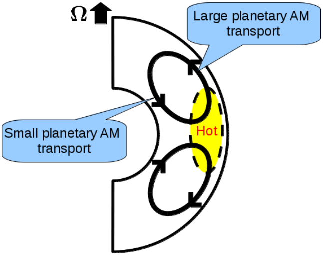
$\Rightarrow$ High temp. in the upper layer around the equator

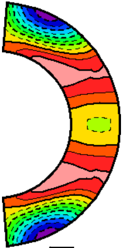
$\Rightarrow$ Latitudinal temperature gradients

$\Rightarrow$ Induced mean meridional circulation

$\Rightarrow$ Transport stellar AM from the equator

$\Rightarrow$ Equatorial sub-rotation





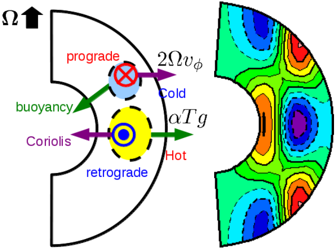
$E_k = 1/\sqrt{10}, P_r = 10^2$

Rapidly rotating cases with large P_r

Thermal wind balance

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = -\frac{1}{\rho}\nabla p - \alpha T \boldsymbol{g} + \nu \nabla^2 \boldsymbol{v}$$

- Taking rotation,
$$-(2\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{v} = -\alpha \nabla T \times \boldsymbol{g}$$
- Longitudinal components:
$$2\Omega \frac{\partial v_\phi}{\partial z} = -\frac{\alpha g(r)}{r}\frac{\partial T}{\partial \theta}$$
- Thermal wind balance :
  - Torque by Coriolis force = Torque by buoyancy



$E_k = 10^{-3}, P_r = 10^2$

Rapidly rotating cases with small P_r

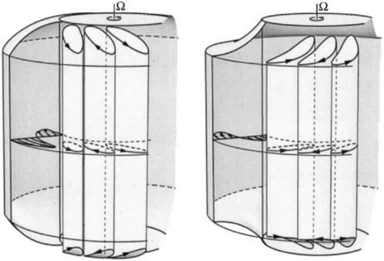
- Curvature of the outer boundary

$\Rightarrow$ tilting of convection cells

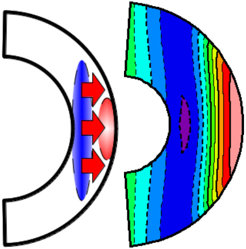
$\Rightarrow$ velocity correlation $\overline{v'_\phi v'_r}$

$\Rightarrow$ Reynolds stress transports AM outward

$\Rightarrow$ equatorial super-rotation (cf. Busse 1983)



(Busse 2002)



$E_k = 10^{-3}, P_r = 1$

Slowly rotating cases with small P_r

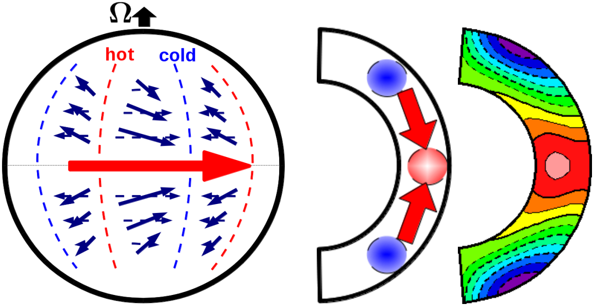
- Sectorial type horizontal structure

$\Rightarrow$ longitudinal velocity is twisted by Coriolis

$\Rightarrow$ velocity correlation $\overline{v'_\phi v'_\theta}$

$\Rightarrow$ Reynolds stress transports AM equatorward

$\Rightarrow$ equatorial super-rotation (cf. Busse 1970, 1973)

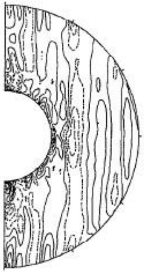


$E_k = 1/\sqrt{10}, P_r = 1$

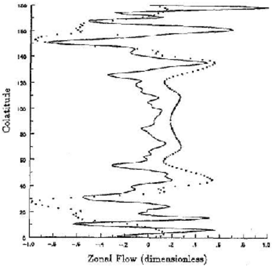
Outline

Pioneering calculations

Radial velocity



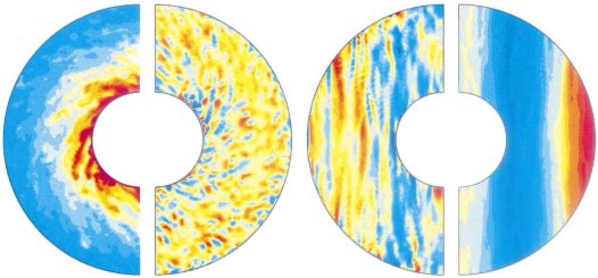
Mean zonal flows



Surface zonal flows

- Sun and Schubert (1995)
    - Numerical calculations with large Ra, low Ek, high resolution
    - Alternating band structure emerges?

Higher resolution, longer time integrations

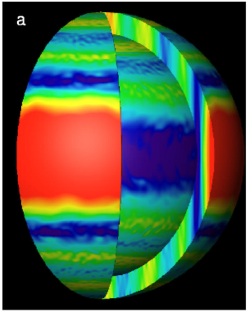


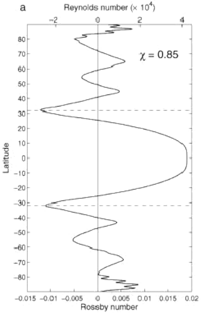
Temperature Axial vorticity (equatorial, meridional) Mean zonal flows

- Christensen (2002)
  - Systematic parameter study for high-Rayleigh number convection
  - Alternating band structure do not emerge!
  - Interpret alternating bands of SS1995 as residual of initial conditions

“Thin” spherical shell model

- Heimpel and Aurnou(2007) (hereafter, HA2007)
  - “Thin” spherical shell model with large Rayleigh number, small Ekman number.
  - Prograde jets and alternating jets in mid- and high-latitudes can produce simultaneously
  - However, eight-fold symmetry in the longitudinal direction is assumed.





Outline

Purpose

- HA2007: eight-fold symmetry in the longitudinal direction is assumed.
  - The artificial limitation of the computational domain may influence on the structure of the global flow field.
    - Zonal flows may not develop efficiently due to the insufficient upward cascade of two-dimensional turbulence
    - Stability of mean zonal flows may change with the domain size in the longitudinal direction.
- In the present study:
  - Numerical simulations in the whole thin spherical shell domain.
  - Coarse spatial resolution and slow rotation rate are used due to the limit of computational resources.

Experimental setup

- Boundary condition: Isothermal, Impermeable and Stress free.
- Input parameters:

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: η	0.75	$0.85, 0.9$
Ekman number: E	10^{-4}	10^{-6}
Modified Rayleigh number: Ra^*	$0.05, 0.1$	0.05

  - the definition of modified Rayleigh number: $\mathrm{Ra}^* = \dfrac{\mathrm{RaE}^2}{\mathrm{Pr}} = \dfrac{\alpha g \Delta T D}{\Omega^2 D}$
    - the ratio of Coriolis term and buoyancy term
- Output parameters:
  - (local) Reynolds number, Re, is equivalent to the non-dimensional velocity in the chosen scaling.
  - (local) Rossby number: $\mathrm{Ro} = \mathrm{ERe}$

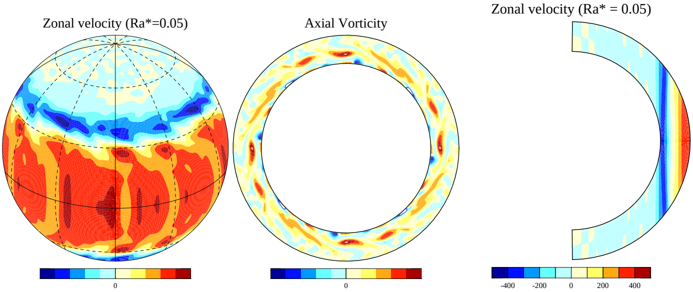
Numerical methods

- Traditional spectral method.
  - Toroidal and Poloidal potentials of velocity are introduced.
  - The total wave number of spherical harmonics is truncated at 170, and the Chebychev polynomials are calculated up to the 48th degree.
    - The numbers of grid points:512, 256, and 48 in the longitudinal, latitudinal, and radial directions, respectively.
- In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies

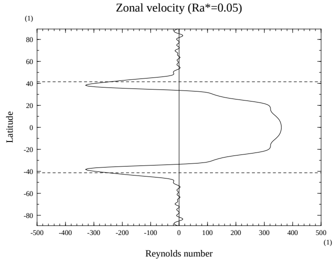
$$\nu = \begin{cases} \nu_0, & \text{for } \; l \leq l_0, \\ \nu[1 + \varepsilon(l - l_0)^2], & \text{for } \; l > l_0. \end{cases}$$

  - we choose $l_0 = 85, \varepsilon = 10^{-2}$
- The time integration is performed using the Crank-Nicolson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

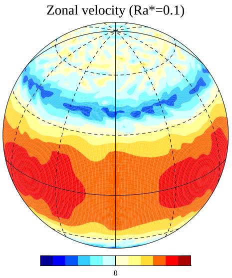
Results: Ra* = 0.05

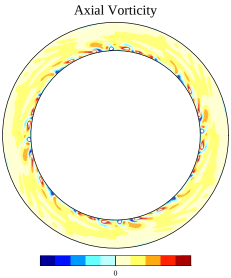


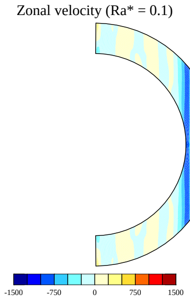
- Broad prograde equatorial jet
- Alternating zonal jets do not emerge in mid- and high-latitudes.



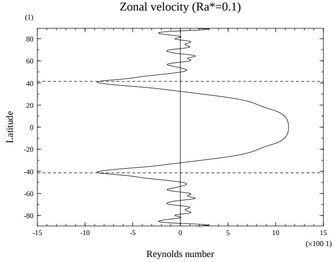
Results: $Ra^* = 0.1$







- Broad prograde equatorial jet
- Mean zonal flow: alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?



Results: output parameters

- Input parameters

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: η	0.75	$0.85, 0.9$
Ekman number: E	10^{-4}	10^{-6}
Modified Rayleigh number: Ra^*	$0.05, 0.1$	0.05

- Output parameters

parameters	present study	HA2007
local Reynolds numnber: Re	$3.59 \times 10^2, 1.13 \times 10^3$	5×10^4
local Rossby number: Ro	$3.59 \times 10^{-2}, 1.13 \times 10^{-1}$	$1.2 \times 10^{-2}, 2.5 \times 10^{-2}$

  - small Re, i.e. weak jet
  - Ro, i.e. slow rotation rate

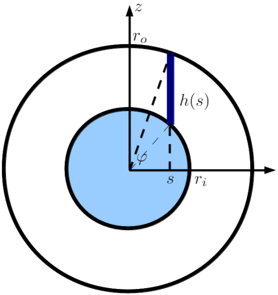
Comparison with the Rhines scale

- Rhines wavenumber and wavelength :
  $k_\beta = \sqrt{\beta/2U}$, $\lambda_\beta = 2\pi\sqrt{2U/\beta}$.
- Inside the tangent cylinder, assuming 2-dim columnar motion,

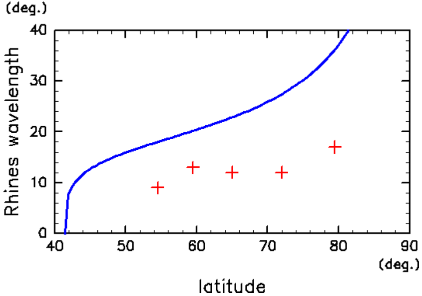
$$\beta = \frac{2}{\mathrm{E}}\frac{1}{h}\frac{dh}{ds}, \quad h(s) = \sqrt{r_o^2 - s^2} - \sqrt{r_i^2 - s^2}.$$

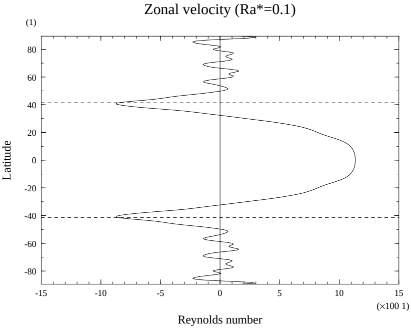
- Substituting $s = r_o \cos\varphi$, we can get jet latitudinal wavelength φ_β
  (Heimpel and Aurnou 2007).

$$\varphi_\beta \equiv \frac{\lambda_\beta}{r_o \sin\varphi} = 2\pi\sqrt{\frac{U\mathrm{E}(\chi^2 - \cos^2\varphi)}{r_o \sin\varphi \cos\varphi}}.$$



Comparison with the Rhines scale





Blue line : $\varphi_\beta(\varphi)$, red cross : jet wavelength from the numerical result

Rhines scale can explains the jet wavelength (qualitatively?)

Outline

Summary and discussion

Summary

- the profile of mean zonal flow:
  - Broad prograde equatorial jet
  - Alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?
  - thick shell? , small Ra^* ?, large E ?
  - hyperdiffusivity?

In the future...

- More 'thin', 'fast rotating' spherical shell convection
  - $\eta = 0.75$ $\eta = 0.8, 0.85, 0.9$
  - $\mathrm{E} = 10^{-4} \rightarrow \mathrm{E} = 3 \times 10^{-5}$
- Investigation of generation mechanism

References

[1] Busse, F. H., 1970a : Differential rotation in stellar convection zones. *Astrophys. J.*, **159**, 629–639

[2] Busse, F. H., 1970b : Thermal instabilities in rapidly rotating systems. *J. Fluid Mech.*, **44**, 441–460

[3] Busse, F. H., 1973 : Differential rotation in stellar convection zones. II. *Astron. Astrophys.*, **28**, 27–37

[4] Busse, F. H., 1983 : A model of mean zonal flows in the major planets. *Geophys. Astrophys. Fluid Dyn.*, **23**, 153–174

[5] Busse, F. H., 2002 : Convective flows in rapidly rotating spheres and their dynamo action. *Phys. Fluids*, **14**, 1301–1314

[6] Christensen, U. R., 2002 : Zonal flow driven by strongly supercritical convection in rotating spherical shells. *J. Fluid Mech.*, **470**, 115–133.

[7] Heimpel, M., Aurnou, J., 2007 : Turbulent convection in rapidly rotating spherical shells: A model for equatorial and high latitude jets on Jupiter and Saturn *Icarus*, **187**, 540–557

[8] Sun, Z.-P., Schubert, G., 1995 : Numerical simulations of thermal convection in a rotating spherical shell at high Taylor and Rayleigh numbers., *Phys. Fluids*, **7**, 2686–2699.

[9] Takehiro, S., Hayashi, Y.-Y., 1999 : Mean zonal flows excited by critical thermal convection in rotating spherical shells. *Geophys. Astrophys. Fluid Dyn.*, **90**, 43–77

[10] Takehiro, S., Irie, M., 2001 : Two-dimensional spiraling convection in a rapidly rotating cylindrical annulus and the dynamics of Rossby waves. *NAGARE Multimedia*, `http://www2.nagare.or.jp/mm/2001/takehiro/index.htm`

[11] Takehiro, S. 2008 : Physical interpretation of spiralling-columnar convection in a rapidly rotating annulus with radial propagation properties of Rossby waves. *J. Fluid Mech.*, **614**, 67–86.

[12] Takehiro, S., 2010 : On the retrograde propagation of critical thermal convection in a slowly rotating spherical shell. *J. Fluid Mech.*, **659**, 505–515

[13] Zhang, K., 1992 : Spiralling columnar convection in rapidly rotating spherical fluid shells. *J. Fluid Mech.*, **236**, 535–556

Taylor Proudman theorem

- Coriolis term dominates in the momentum eq. ⇒ geostrophic balance

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = -\frac{1}{\rho}\nabla p - \alpha T \boldsymbol{g} + \nu \nabla^2 \boldsymbol{v}$$

- Taking rotation operation,

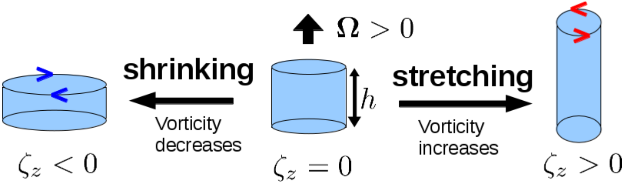
Taylor Proudman theorem

$$(2\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{v} = 0.$$

- Fluid motion is uniform along the rotation axis ⇒ two-dimensional motion (e.g. Taylor columns)

Conservation of potential vorticity

- Conservation of potential vorticity ⇔ Local conservation of angular momentum



Conservation of potential vorticity

$$\frac{2\Omega + \zeta_z}{h} = \text{const.}$$

where ζ_z is the axial vorticity component, h is the hight of the fluid column.