Mantle convection: complex rheology

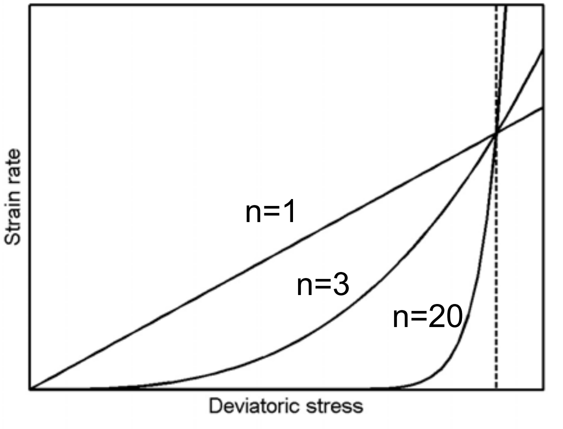
While simple convection models assume constant viscosity, the mantle rheology depends strongly on temperature and pressure through an Arrhenius law, and on deviatoric stress and grain size d.

$$\dot{\boldsymbol{\varepsilon}} = A\,\boldsymbol{\tau}\,\tau^{n-1}\,d^{-m}\exp\left(-\frac{E^* + pV^*}{RT}\right)$$

$\dot{\varepsilon}_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is strain rate, $\tau_{ij} = \sigma_{ij} + p\delta_{ij}$ deviatoric stress, E^* is activation energy and V^* is activation volume. The (effective) viscosity is $\eta = \tau/\dot{\varepsilon} = \eta(p,T,\tau,d)$.

Laboratory experiments on rock deformation at high temperature show:

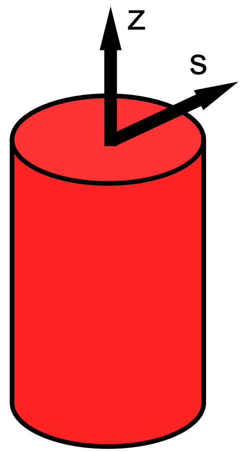
(1) $E^* \approx 300 - 550$ kJ/mol. Temperature increase by 100° $\Rightarrow$ viscosity decrease factor 10.
(2) V^* not well constrained, but viscosity increase in mantle by factor $10^2 - 10^4$ plausible.
(3) At low stress, n = 1 (Newtonian viscosity), but m ≈ 2 (grain size dependence).
(4) At high stress, n > 1, typically n=3-5 (power-law creep), m=0.
(5) At low T and very high stress, n>>1. In the extreme limit ($n \to \infty$) this corresponds to a yield-stress rheology, where $\dot{\varepsilon}=0$ for $\tau<\tau_{yield}$ and $\dot{\varepsilon}$ arbitrary for $\tau=\tau_{yield}$.
(6) Several different creep mechanisms may operate at the same time. The individual strain rates are added.



Symbols: (Bold symbols are tensors): $\dot{\varepsilon}$ – strain rate, τ –deviatoric stress, σ – full stress, δ_{ij} – Kronecker symbol, A – constant, d – grain size, E*, V* - activation energy (volume), R – gas constant, T –absolute temperature

Conduit plumes I

In convection with strongly temperature-dependent viscosity, the flow rising from the hot bottom boundary layer takes the form of narrow, approximately cylindrical plumes.



Assumptions: Axisymmetric ($\partial/\partial\varphi=0$), Boussinesq, steady state ($\partial/\partial t=0$), narrow ($\partial/\partial s >> \partial/\partial z$, $w>>u_s$) plume. Use z-component of Navier-Stokes eqn. in cylindrical coordinates (viscous term generalized for $\eta\neq$const); $\partial P/\partial z=0$: local balance between buoyancy and shear forces τ_{sz}. Define $\theta=(T-T_\infty)/\Delta T$.

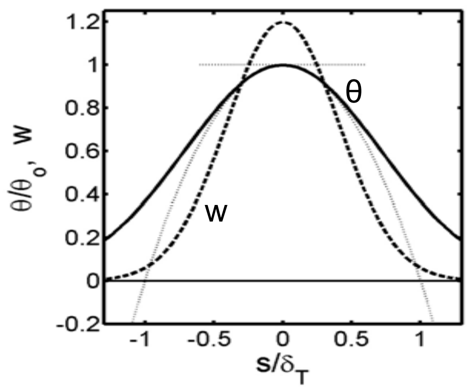
$$\rho\,\alpha\,g\,\Delta T\,\theta + \frac{1}{s}\frac{\partial}{\partial s}\left(s\,\eta\,\frac{\partial w}{\partial s}\right) = 0$$

$$w\frac{\partial\theta}{\partial z} = \frac{\kappa}{s}\frac{\partial}{\partial s}\left(s\frac{\partial\theta}{\partial s}\right)$$

$$\eta = \eta_\infty \exp(-\gamma\theta)$$

Boundary conditions: $\theta=1$ at $z=0$; $\theta=0$ for $z\rightarrow\infty$, $s\rightarrow\infty$; $w=0$ for $s\rightarrow\infty$; $\partial\theta/\partial s=\partial w/\partial s=0$ at $s=0$.

To find velocity profile, use simplifying assumptions **$\theta(s,z) = \theta_o(z)\,(1-[s/\delta_T]^2)$**, to give **$\eta=\eta_o(z)\exp(\gamma\theta_o[s/\delta_T]^2)$** with **$\eta_o(z)=\eta_\infty\exp(\gamma\theta_o)$** and ignore s-dependence ($\theta=\theta_o$) in buoyancy term. Integrate twice to obtain:



$w(s,z) = w_o(z)\exp(-[s/\delta_w]^2)$ with $\delta_w=\delta_T(\gamma\theta_o)^{-1/2}$ and $w_o(z) = \rho\alpha g\Delta T\delta_T^2/(4\gamma\eta_o)$.

Symbols: s radial coordinate, θ – non-dim. temperature, T_∞ - temperature of ambient mantle, ΔT – temperature contrast at z=0, η_∞ - viscosity of ambient mantle, γ – viscosity parameter, δ_T (δ_w) – width of temperature (velocity) anomaly

Conduit plumes II

To obtain equation for change of axial temperature with height, evaluate the temperature equation at s=0:

$$\frac{d\theta_o}{dz} = -\frac{4\kappa\theta_o}{w_o\delta_T^2} \qquad (I)$$

Another relation between w_o and θ_o comes from the requirement of constant **buoyancy flux** B = ∬δρwdS:

$$B = \rho\alpha\Delta T\int_0^\infty 2\pi\, s\, w\,\theta\, ds \approx \frac{\pi\rho\alpha\Delta T\delta_T^2 w_o}{\gamma} = const \qquad (II)$$

Eliminating $w_o\delta T^2$ from (I) and integrating, and eliminating w_o from (II), we obtain:

$$\theta_o(z) = \exp\left(\frac{-4\pi\kappa\rho\alpha\Delta T z}{B\gamma}\right) \qquad \delta_T = \left(\frac{2\gamma}{\rho\alpha\Delta T}\right)^{1/2}\left(\frac{B\eta_o}{\pi g}\right)^{1/4} \qquad w_o = \left(\frac{gB}{4\pi\eta_o}\right)^{1/2}$$

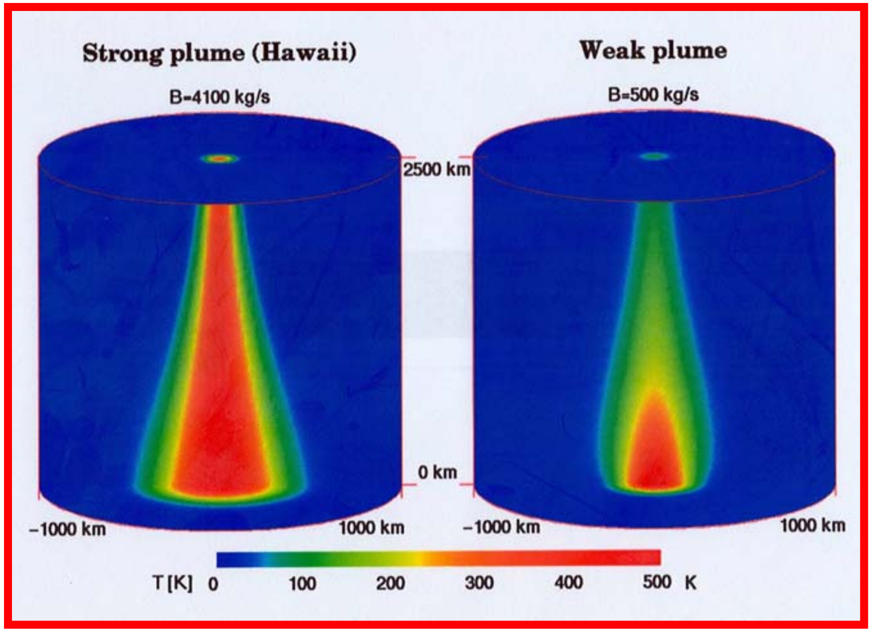
(1) The heat flux is given by $Q = Bc_p/\alpha$ and the volume flux by $dV/dt \approx B/(\rho\alpha\Delta T)$.
(2) The axial temperature decreases exponentially with height, scale height is $z_\theta \sim B$.
(3) Like in Poiseuille-flow $dV/dt \sim B \sim \delta_T^4 / \eta$.
(4) For realistic $\gamma>1$, $\theta_o \approx 1$, thermal anomaly δ_T is wider than region of fast upwelling δ_w

Symbols: B – buoyancy flux, dS – area element, c_p – heat capacity, Q – heat flux, dV/dt – volume flux, z_θ – temperature scale height.

Application to mantle plumes

Use typical values in the upper mantle: ρ=3300 kgm^{-3}, α=3x10^{-5} K^{-1}, ΔT=300 K (from melting at hot spots), η_∞=3x10^{20} Pa s, γ=6, κ=10^{-6} m^{2}s^{-1}, B=4000 kgs^{-1} or 500 kgs^{-1} (strong plume or weak plume, estimated from hot-spot swell). Assume that plume comes from core-mantle boundary (z≈2800 km at bottom of lithosphere):

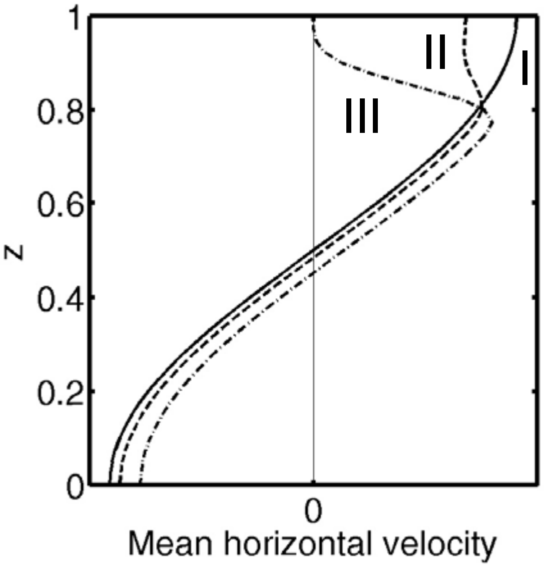
(1) $\theta_o \approx 0.96 / 0.70$ (strong / weak plume). Strong plume has hardly cooled in center, plumes much weaker than B=500 kgs^{-1} would fade away while rising through the mantle.
(2) η_o = 10^{18} Pa s / 4.5x10^{18} Pa s.
(3) δ_T = 67 km / 58 km. Weak plume hardly narrower.
(4) w_o = 180 cm yr^{-1} / 30 cm yr^{-1}.
(5) If background viscosity η_∞ increases with depth, plume becomes slimmer with height

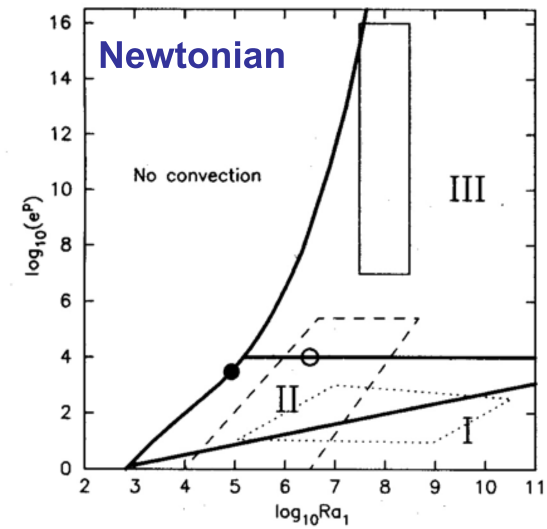


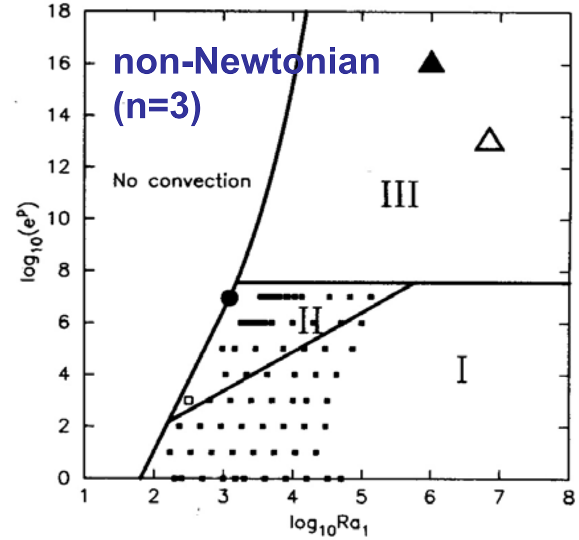
Numerical model of strong and weak plume

Convection modes for strongly variable η

In the upper boundary layer of convection, the viscosity may increase by many orders of magnitude if it is strongly temperature-dependent. The surface layer becomes a stagnant lid.







Three regimes:

(I) Mobile lid, nearly like isoviscous convection, at moderate viscosity contrast and high Rayleigh number

(II) Transitional (sluggish lid), at moderate viscosity contrast and lower Rayleigh number

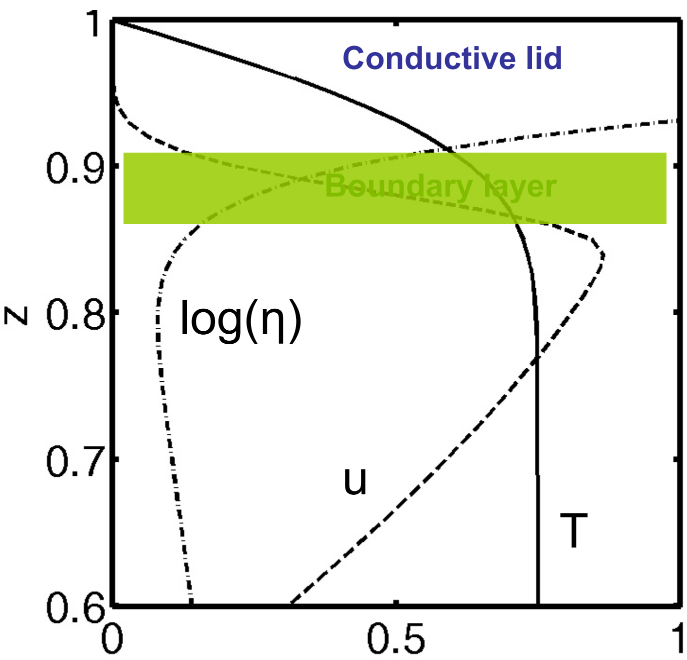
(III) Stagnant lid, at high viscosity contrast (≈10^4 for n=1 and ≈10^8 for n=3)

In planetary mantles, the viscosity contrast between surface and interior is enormous!

Stagnant lid convection

In stagnant lid convection, heat is transported by conduction in most of the top layer. The convectively unstable bottom part is restricted to the region, which is (1) significantly colder than the interior, but (2) not so cold that it is too stiff to participate in the convection. The second criterion typically means that its viscosity exceeds that of the interior by not more than a factor of ≈10. This is equivalent to a temperature drop of 100° – 200° across the convective boundary layer (compared to ≈1000° across the conductive lid).

All planets except Earth seem to operate at present in the stagnant lid regime.



Thermal evolution of convecting planets

The internal heat of a planet inherited from the accretion process and produced by radioactive trace elements is transported by convection and lost through the surface. In general the different parts of a planet will cool with time. Convection in the silicate mantle is more sluggish than in the (fluid) core, therefore mantle dynamics controls the cooling.

Approach of parameterizing convective heat loss:

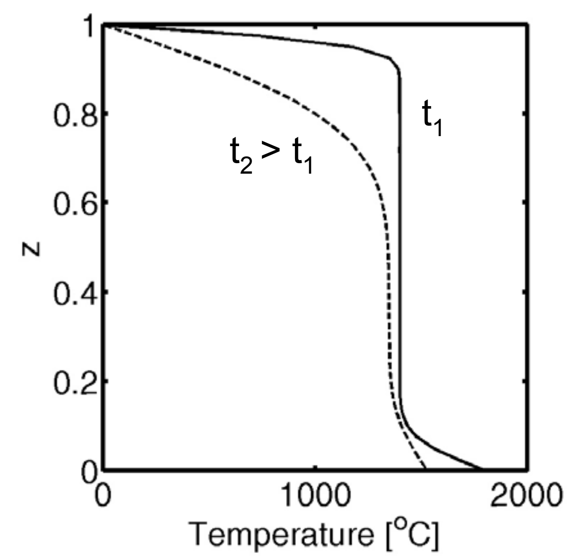
Conservation of energy: $C \, dT_m/dt = H(t) - Q(T_m)$

Assumption: $Q \sim T_m \, Nu \sim T_m \, Ra^{\beta} \sim T_m^{(1+\beta)} \exp(-\beta E^*/RT_m)$ where $\beta \approx 0.3$

Predictions of simple parametrized models:

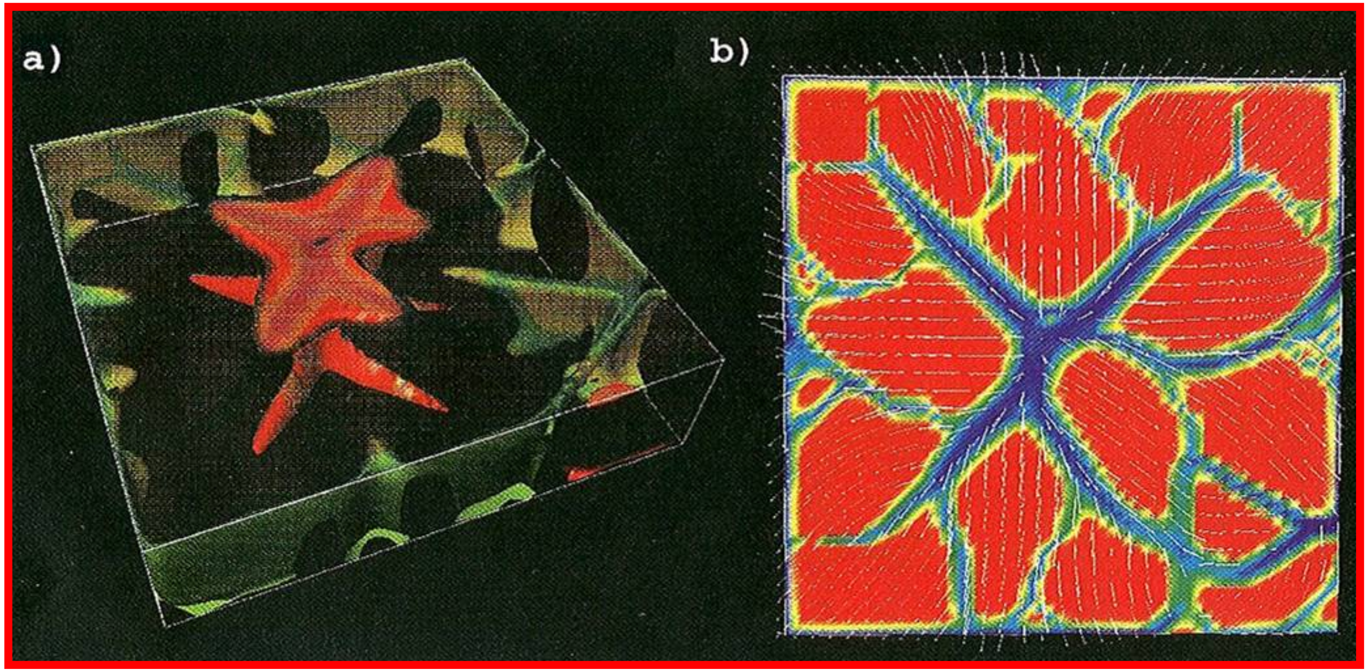
- Moderate reduction of mantle temperature with time
- Strong reduction of convective vigour and heat flow
- Heat loss and heat production not far out of equilibrium (Urey ratio $H/Q \approx 0.7 - 0.9$)

This approach is of limited value: it ignores that (in the stagnant lid regime) heat is transported by convection only to the bottom of the lid, through which it must be conducted. Numerical convection models show that planets with a stagnant lid cool mainly by growing their lithosphere. The temperature in the convecting region decreases only slightly.



Symbols: C – heat capacity of planet, T_m – mean mantle temperature, H – heat production, Q – heat loss through the surface, β – exponent relating Nusselt number to Rayleigh number

Plate tectonic convection



Tackley, EPSL, 1998
Combination of Newtonian viscosity (η=1 in mantle, $\eta=10^4$ in lid) and yield-stress rheology. Flow driven by buoyancy taken from a convection model (no self-consistent evolution).
a) Temp. isosurfaces
b) Surface motion (arrows) and viscosity (blue=low, red=high)

With a yield-stress rheology in the lid it is possible to simulate mobile surface plates. The effective viscosity is high in the interior of plates and low at plate margins, where most of the deformation is concentrated. The plate interior moves at uniform velocity.
Full 3D-convection models [e.g. Stein et al., PEPI, 2004] that combine T-dependent viscosity and yield-stress rheology show plate-like behaviour only in a narrow parameter range and differs in some respect from terrestrial plate tectonics (e.g. no one-sided subduction).
$\Rightarrow$ Mantle rheology even more complex. Role of water for „lubricating“ plate subduction may explain difference between Earth and Venus.