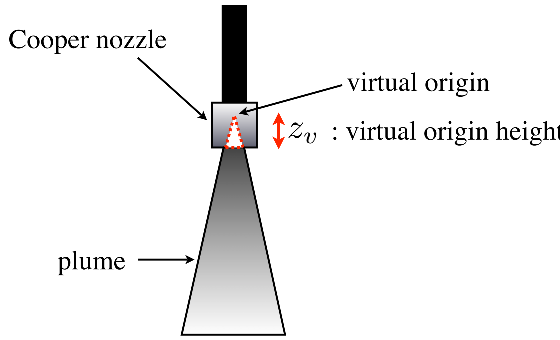

Figure 10: Schematic illustration of virtual origin.

The filling tank experiment is a good method to determine both the virtual origin height and the entrainment constant.

A.1 Theory of the filling tank experiment

In this section, the theory of filling tank experiment (Baines and Turner, 1969) is explained. First, some general properties of an axisymmetric turbulent plume are derived. Then the method of determining both the virtual origin height and the entrainment constant is explained.

A.1.1 General properties of an axisymmetric turbulent plume

It is assumed that the profiles of vertical velocity w and buoyancy g' of the plume are of Gaussian form,

$$w(z,r) = W(z)\exp\left(-\frac{r^2}{b(z)^2}\right), \tag{6}$$

$$g'(z,r) = g\left(\frac{\rho_A - \rho_P(z,r)}{\rho_R}\right) = G'(z)\exp\left(-\frac{r^2}{b(z)^2}\right). \tag{7}$$

Here z is vertical coordinate, r the radial distance from the axis of the plume, g the gravity, $W(z) = w(z,0)$, and $G'(z) = g'(z,0)$. Then ρ_A and ρ_P are densities of the ambient fluid and the plume, respectively, ρ_R is some standard reference density for the system. Note that b is the radius where buoyancy and vertical velocity are reduced by a factor $1/e$ from those on the plume's axis.

Volume flux Q of the axisymmetric turbulent Gaussian plume is

$$Q = \int_0^{2\pi}\int_0^{\infty} wrdrd\theta = \pi b^2 W, \tag{8}$$

where θ is angular coordinate.

The momentum flux M is

$$M = \int_0^{2\pi}\int_0^{\infty} w^2 rdrd\theta = \frac{\pi}{2}b^2W^2. \tag{9}$$