For non-interacting plumes (i.e. $x(z) = x_0$), (36) is easily solved to give

$$x_0 = \sqrt{2} b_m. \tag{37}$$

Here the subscript m denotes the value at the point where plumes merge. Dividing (37) by x, we obtain

$$\gamma_m = \frac{1}{\sqrt{2}}, \tag{38}$$

so, in terms of the non-dimensional height we obtain

$$\hat{\lambda}_m = \frac{1}{\sqrt{2}} \frac{5}{6\alpha}. \tag{39}$$

Here $(\hat{\cdot})$ denotes the upper bound on the value. As shown by Bjorn and Neilsen (1995) this estimate of λ_m is poor. In order to model the drawing together of two equal plumes we need to consider the entrainment of one plume by another.

Based on experimental results (e.g. Rouse, Yih, and Humpherys, 1952) it is reasonable to take the velocity field outside the plumes, created by entrainment, as horizontal. The mean entrainment velocity field, over a horizontal plane across the two plumes, may be approximated by two sinks of strength $-m(z)$ placed at $(-\frac{1}{2}x, z)$ and $(\frac{1}{2}x, z)$. The complex velocity potential in this horizontal plane is given by

$$\Psi = -\frac{m}{2\pi} \left[\ln \left(Z + \frac{1}{2} x \right) + \ln \left(Z - \frac{1}{2} x \right) \right], \tag{40}$$

where Z is the complex variable $re^{i\theta}$. The velocity field is given by

$$U = \frac{\partial \Psi}{\partial Z} = -\frac{m}{2\pi} \left(\frac{1}{Z + \frac{1}{2}x} + \frac{1}{Z - \frac{1}{2}x} \right). \tag{41}$$

The sink strength of the plume is

$$m = \int_0^{2\pi} b\alpha w d\theta. \tag{42}$$

Along the line joining the sources of the plume ($\theta = 0$) the velocity is given by

$$U_{\theta=0} = -b\alpha w \left(\frac{1}{r + \frac{1}{2}x} + \frac{1}{r - \frac{1}{2}x} \right). \tag{43}$$

On the plume axis ($r = -\frac{1}{2}x, \frac{1}{2}x$), the value of horizontal entrained velocity due to that plume is zero, and only the term resulting from the other plume needs to be considered. Therefore, (43) gives an expression for the mean horizontal velocity u on the plume axis

$$u = -\frac{b\alpha w}{x}. \tag{44}$$