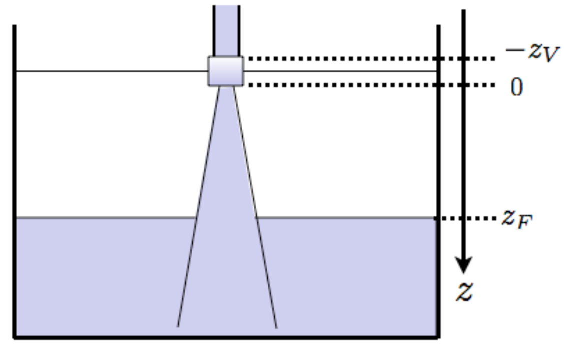


Figure 1: Schematic illustration single plume experiment.

# 3 Analyzing the single plume experiment (Bains and Turner, 1969)

We consider a tank of cross section $A_C$ filled with fresh water, and a axisymetric plume (salt water solution colored by a dye) from the top of the tank ($z = 0$). We set time $t = 0$ when $z_F(t) = z_{F0}$, where $z_F$ is the height of the colored salt water's front which comes up from the bottom. It is convenient to choose $z_{F0}$ at the height where we can measure $z_F$ accurately.

Considering the balance of the volume of the colored salt water at $z = z_F$, time derivative of $(z_F(t) + z_V)$ can be written as

$$\begin{aligned} \frac{\partial}{\partial t}(z_F(t) + z_V) &= -\frac{Q(z_F)}{A_C}, \\ &= -\frac{Q_0}{A_C}(z_F + z_V)^{\frac{5}{3}}. \end{aligned} \tag{22}$$

Integrating (22), we obtain

$$\int_{z_{F0}}^{z_F} (z_F + z_V)^{-\frac{5}{3}} d(z_F + z_V) = -\frac{Q}{A_C}\int_0^t dt \tag{23}$$

$$-\frac{3}{2}\left[(z_F + z_V)^{-\frac{2}{3}} - (z_{F0} + z_V)^{-\frac{2}{3}}\right] = -\frac{Q_0}{A_C}t, \tag{24}$$

$$(z_F + z_V)^{-\frac{2}{3}} = \frac{2}{3}\frac{Q_0}{A_C}t + (z_{F0} + z_V)^{-\frac{2}{3}}. \tag{25}$$

# 4 Coalescing axisymmetric turblent equal plumes (Kaye and Linden, 2004)

We define dimensionless variables as followings,

$$\lambda = \frac{z}{\xi_0}, \quad \phi = \frac{\xi}{\xi_0}, \quad \gamma = \frac{b}{\xi}, \tag{26}$$