Note

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1 Introduction

2 Axisymmetric turbulent Gaussian plume

Volume flux of the axisymmetric turbulent Gaussian plume Q is

$$Q = \int_A w dA = \pi b^2 \overline{w}, \tag{1}$$

where A is the cross section of the plume, w the vertical velocity, $\overline{w}$ the mean vertical velocity, b the radius of the plume.

The momentum flux M is

$$M = \int_A \frac{w^2}{2} dA = \frac{\pi}{2} b^2 w^2. \tag{2}$$

Then b can be written as

$$b = \frac{1}{\sqrt{2}} \pi^{-\frac{1}{2}} \frac{Q}{M^{\frac{1}{2}}}. \tag{3}$$

The buoyancy flux F is

$$F = \int_A \frac{g}{2} \frac{(\rho_A - \rho_P)}{(\rho_A + \rho_P)/2} w dA = \frac{g'}{2} Q, \tag{4}$$

where g is the gravity, $g' = g\frac{(\rho_A - \rho_P)}{(\rho_A + \rho_P)/2}$ the reduced gravity, ρ_A and ρ_P are the density of ambient fluid and the plume, respectivily.

There is an entrainment from the ambient fluid to the plume, and its velocity u_e is now assumed as

$$u_e = -\alpha w, \tag{5}$$

where α is called entrainment constant.

The volume flux Q increases with z, because of the entrainment, so

$$\begin{aligned} \frac{\partial Q}{\partial z} &= -2\pi b u_e, && (6) \\ &= 2\pi \alpha b w, && (7) \\ &= 2\sqrt{2}\pi^{\frac{1}{2}} \alpha M^{\frac{1}{2}}. && (8) \end{aligned}$$