Eliassen-Palm, Charney-Drazin, and the development of wave, mean-flow interaction theories in atmospheric dynamics

David Andrews

My personal memories...

- I'll describe some of the work I did with Michael McIntyre from 1971-78.
- After ~ 30 years, some aspects may have been erased from my memory!
- But I'll try to explain how our ideas developed...
- ... and set them in context.

Earlier history

- Since the work of Starr (MIT) and others in the 1950s-60s, meteorologists had been analysing atmospheric data in terms of zonal means and ‘waves’ or ‘eddies’, e.g.

$$u(x,y,z,t) = \bar{u}(y,z,t) + u'(x,y,z,t).$$

Zonal mean

Wave

- We can write dynamical equations in terms of mean and wave terms.
- Take quasi-geostrophic zonal-mean momentum equation, for simplicity:

$$\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v} - \frac{\partial(\overline{v'u'})}{\partial y}$$

Convergence of ‘eddy momentum flux’

Zonal-mean zonal acceleration

Coriolis term associated with mean meridional circulation

Interpretation?

- We seem to have a nice physical interpretation: mean acceleration is "**due to**"
  a) mean Coriolis term
  b) eddy (or wave) fluxes.
- **BUT** the mean meridional circulation is *not independent* of the eddies/ waves. It may even be forced by them! (See later.)
- Direction of causality is not clear!

‘Cancellation’

- It is sometimes found that the ‘eddy’ and ‘mean’ terms are nearly equal, suggesting that they are somehow related:

$$f_0\bar{v} \approx \frac{\partial(\overline{v'u'})}{\partial y} \gg \left|\frac{\partial\bar{u}}{\partial t}\right|$$

$\Rightarrow$ acceleration is a small residual.

- The full QG set of equations is

Subscripts = partial derivatives

Eddy heat flux

$$\bar{u}_t - f_0 \bar{v} = -(\overline{v'u'})_y \ ,$$

$$\bar{\theta}_t + \theta_{0z}\bar{w} = -(\overline{v'\theta'})_y + \bar{Q} \ ,$$

$$\bar{v}_y + \rho_0^{-1}(\rho_0 \bar{w})_z = 0 \ ,$$

$$f_0 \bar{u}_z = -RH^{-1} e^{-\kappa z/H} \bar{\theta}_y \ .$$

We can solve for $\bar{u}_t$, $\bar{\theta}_t$, $\bar{v}$ and $\bar{w}$, given the eddy terms, the mean heating $\bar{Q}$ and suitable initial and boundary conditions.

- Looking at only one equation (e.g. the zonal momentum equation) can be misleading!
- Eddy fluxes also appear in the zonal-mean thermodynamic equation.

To get $\bar{u}_t$, $\bar{\theta}_t$, $\bar{v}$ and $\bar{w}$, we must solve *complete set* of equations + boundary conditions.

By the early 1970s several theoretical studies had looked at wave-mean interaction in the stratosphere:

- Matsuno (1971): stratospheric sudden warmings, mean-flow acceleration driven by Rossby waves.
- Lindzen & Holton (1968), Holton & Lindzen (1972): quasi-biennial oscillation (QBO), mean-flow acceleration driven by equatorial Kelvin and Rossby-gravity (Yanai) waves.

How did I get involved?

- In 1971, I started a PhD at Cambridge with Michael McIntyre

Me in 1974

- Michael was (among other things) very interested in some wave, mean problems, and had (I think) recognised the importance of *wave transience* and *wave dissipation* in driving mean-flow changes.
- He suggested I should look at the $O(\text{amplitude}^2)$ effect of various waves on mean flows, using a “two-timing” technique, etc.
- I also looked at Lagrangian means, proposed by F. P. Bretherton (1971).
- All this was entirely analytical – no computers were used!
- The most interesting application was to the interaction of Kelvin and RG waves to the QBO (later published in *JAS* **33**, 2049-53, 1976)

- Another important influence on me was Jim Holton, who had a year’s sabbatical in Cambridge while I was doing my PhD.

James R Holton (1938-2004)

Eliassen and Palm (1961)

- I read this famous paper while I was a student…

Arnt Eliassen, 1915-2000

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GEOPHYSICA NORVEGICA

VOL. XXII NO. 3

ON THE TRANSFER OF ENERGY IN STATIONARY MOUNTAIN WAVES

BY ARNT ELIASSEN AND ENOK PALM

FREMLAGT I VIDENSKAPS-AKADEMIETS MØTE DEN 9DE DESEMBER 1960

Summary. The flow of "wave energy" in stationary, two-dimensional gravity waves of small amplitude in a basic current where the velocity and stability varies with height, is studied. The vertical flux of wave energy is found to vary with height in proportion to the wind speed. In layers where the wave motion of a particular wave length is of the internal type, the motion may be subdivided into two parts, one wave carrying wave energy upward, and the other carrying wave energy downward. In the case of mountain waves, the wave with upward energy flow may be interpreted as the incident wave, set up by the mountain, whereas the wave carrying energy downward is caused by reflection of the incident wave in higher layers in the atmosphere. Such reflection is generally found to take place when wind or stability varies with height. The reflection coefficients in two- and threelayer atmospheres are calculated.

The results are applied to a distribution of wind and stability typical of situations in which mountain waves occur. It was found that, depending on the wave length, 65—100 per cent of the wave energy was reflected from the layers of strong wind in the upper troposphere. In middle latitudes in winter, wave energy may be transmitted to the lower ionosphere.

A study is also made of the energy transfer for long quasi-static mountain waves.

- ... but I didn’t fully understand it at the time!
- Near the end of this paper was a section on general steady, non-dissipated waves in a zonal mean flow.
- It gave some mysterious relations between “energy fluxes”, “momentum fluxes” and “heat fluxes” associated with the waves...

Multiplication of this equation by $[Uu + \varphi - \sigma^{-1} UU_p\varphi_p]$ and averaging gives

$$\overline{\varphi v} = U\,[\sigma^{-1} U_p\, \overline{v\varphi_p} - \overline{uv}] \tag{10.5}$$

Next we multiply (9.4) by $(Uu + \varphi)$ and average:

$$(f - U_y)\,(U\overline{uv} + \overline{\varphi v}) = U_p\,(U\overline{u\omega} + \overline{\varphi\omega}) \tag{10.6}$$

Elimination of $\overline{\varphi v}$ between (10.5) and (10.6) gives

$$\overline{\varphi\omega} = U\,[\sigma^{-1}\,(f - U_y)\,\overline{v\varphi_p} - \overline{u\omega}] \tag{10.7}$$

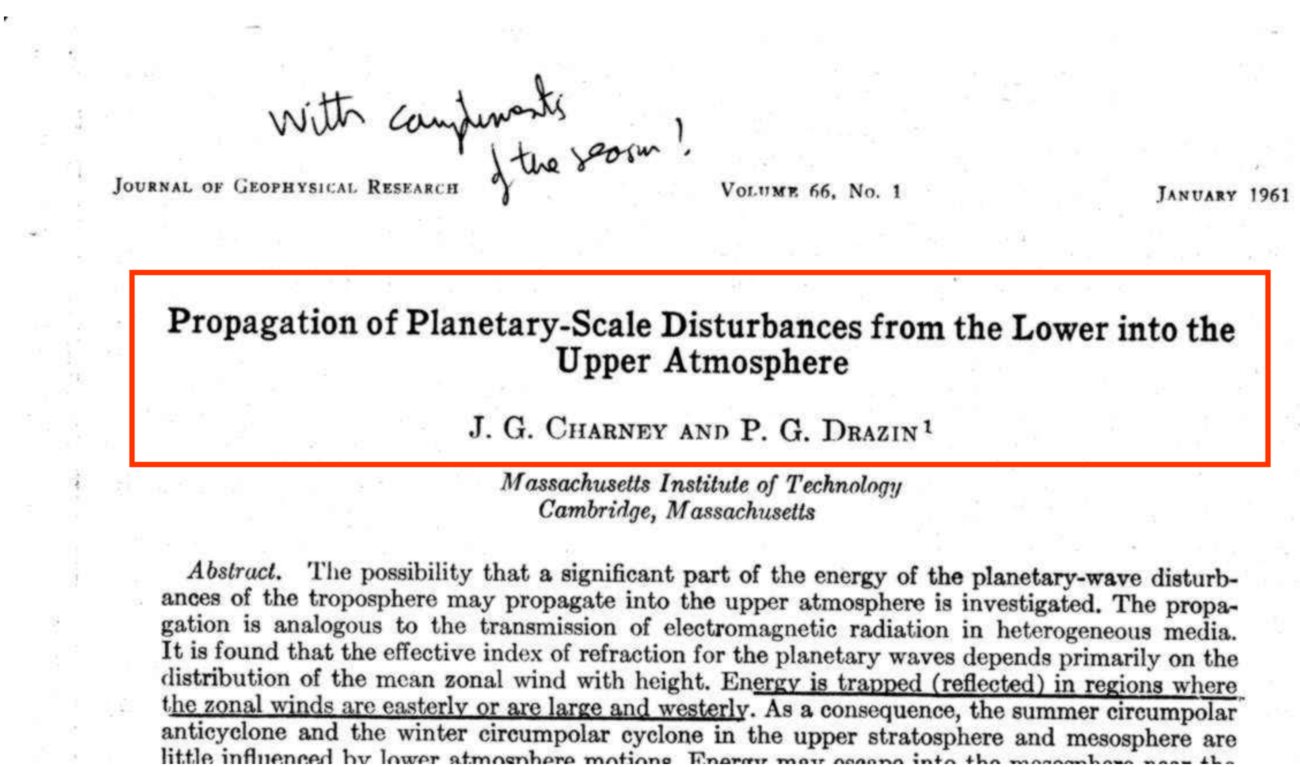
When the expressions (10.5) and (10.7) for the energy fluxes are entered into the energy equation (10.3), we obtain

$$\frac{\partial}{\partial y}[\sigma^{-1}U_p\,\overline{v\varphi_p} - \overline{uv}] + \frac{\partial}{\partial p}[\sigma^{-1}\,(f - U_y)\,\overline{v\varphi_p} - \overline{u\omega}] = 0 \tag{10.8}$$

- I don't think anyone (except possibly Eliassen) really understood at the time what these meant!

Charney and Drazin (1961)

- I also read this important paper:



With compliments of the season!

JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 66, No. 1 JANUARY 1961

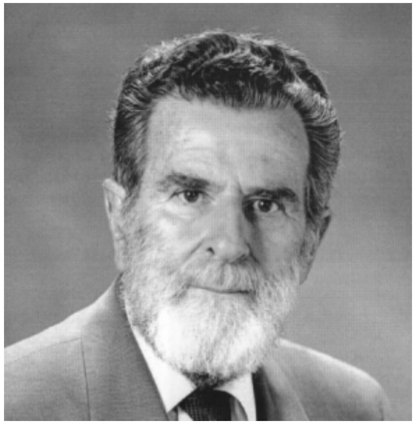
Propagation of Planetary-Scale Disturbances from the Lower into the Upper Atmosphere

J. G. CHARNEY AND P. G. DRAZIN[1]

Massachusetts Institute of Technology
Cambridge, Massachusetts

Abstract. The possibility that a significant part of the energy of the planetary-wave disturbances of the troposphere may propagate into the upper atmosphere is investigated. The propagation is analogous to the transmission of electromagnetic radiation in heterogeneous media. It is found that the effective index of refraction for the planetary waves depends primarily on the distribution of the mean zonal wind with height. Energy is trapped (reflected) in regions where the zonal winds are easterly or are large and westerly. As a consequence, the summer circumpolar anticyclone and the winter circumpolar cyclone in the upper stratosphere and mesosphere are little influenced by lower atmosphere motions. Energy may escape into the mesosphere near the

Jule Charney, 1917-81



Philip Drazin, 1934-2002

- It was most famous for working out the mean-flow conditions under which linear planetary (Rossby) waves can propagate into the upper atmosphere.

- But it also had a section on the nonlinear effects of these waves on the mean flow.

$$\bar{\rho}u_0(f_0^2/N^2)B = \overline{p'w'} \qquad (8.12)$$

and evaluation of the integrals in equation 2.21 for a thin horizontal layer for the case of stationary flow gives

$$\bar{\rho}(du_0/dz)(f_0^2/N^2)B = d\,\overline{p'w'}/dz \qquad (8.13)$$

the first term on the right-hand side of (2.21) vanishing because of the phase shift in $\partial\chi'/\partial x$ and $\partial\chi'/\partial y$. Differentiation of (8.12) and substitution from (8.13) then lead to the result that $\bar{\rho}B/N^2$ and $p'w' - \rho B u_0/N^2$ are independent of height if $u_0 \neq 0$. This result was first obtained by A. Eliassen who communicated it to the authors. It follows that equation 8.5, as well as the boundary and interface conditions, are homogeneous, and we may conclude that $\partial\chi_0/\partial t$ vanishes identically, i.e., that the second-order changes in the zonal flow are zero.

- It showed (following a suggestion by Eliassen) that the steady, non-dissipated waves they considered, had no effect on the mean flow.
- Later this came to be called a non-acceleration theorem.

- Uryu (1973): clarified EP using Lagrangian particle displacements.

86 Journal of the Meteorological Society of Japan Vol. 51, No. 2

On the Transport of Energy and Momentum in Stationary Waves in a Rotating Stratified Fluid

By Michiya Uryu

Department of Physics, Kyushu University, Fukuoka, Japan

(Manuscript received 30 October 1972, in revised form 8 March 1973)

Abstract

The transport properties of stationary waves superposed on a basic zonal current with horizontal and vertical shears in a rotating stratified fluid are discussed.

- Uryu (1974a, 1974b, 1975..): several papers on $O(\text{amplitude}^2)$ mean motions induced by wave packets

August 1974 M. Uryu 34

Induction and Transmission of Mean Zonal Flow by Quasi-geostrophic Disturbances

By Michiya Uryu

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(Manuscript received 5 January 1974, in revised form 2 July 1974)

Abstract

The induction and transmission of mean motion by quasi-geostrophic disturbances are discussed. Two types of momentum transport process, *i.e.*, that by a propagating internal wave packet and that by vortex tube stretching, are discussed by the use of a simple inviscid Boussinesq fluid channel model. The disturbance is excited by transient motion of the corrugated bottom, and the evolution of mean zonal flow is examined.

After finishing my PhD...

- In 1975 I went to work on other problems with Raymond Hide at the UK Met Office and Brian Hoskins at Reading University.
- However, I kept up my interest in wave-mean theory, in particular wondering whether a general theory could be developed that took wave transience and dissipation into account.

Generalisation of EP and CD

- After much algebra, McIntyre and I found that we could generalise the results of Eliassen & Palm and Charney & Drazin.
- EP's mysterious eddy relation (10.8) was shown to be a special case of a "conservation law" for wave properties, valid when the waves are steady and non-dissipated.

Again using QG for simplicity, define EP flux

$$\mathbf{F} \equiv \left(0, -\rho_0 \overline{v'u'},\ \rho_0 f_0 \frac{\overline{v'\theta'}}{\theta_{0z}}\right) .$$

Then, to $O(\text{amplitude}^2)$,

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{F} = D ,$$

where A is a measure of *wave activity* and D is wave dissipation.

The *generalised EP theorem*: a **conservation law** for $O(\text{amplitude}^2)$ wave properties.

Transformed Eulerian-mean formulation

Introduce the *residual circulation*

$$\bar{v}^* \equiv \bar{v} - \rho_0^{-1}\left(\frac{\rho_0\overline{v'\theta'}}{\theta_{0z}}\right)_z \,, \qquad \bar{w}^* \equiv \bar{w} + \left(\frac{\overline{v'\theta'}}{\theta_{0z}}\right)_y \;;$$

then zonal-mean momentum equation becomes

$$\frac{\partial \bar{u}}{\partial t} - f_0\bar{v}^* = \rho_0^{-1}\nabla\cdot\mathbf{F}$$

and there is no 'eddy forcing' in the zonal-mean thermodynamics equation

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{w}^*\theta_{0z} = \bar{Q}\,.$$

The EP flux divergence is the *only* eddy term here.

- So the eddy heat and momentum fluxes do not act separately, but in the combination

$$\nabla \cdot \mathbf{F} \equiv \frac{\partial}{\partial y}\left(-\rho_0 \overline{v'u'}\right) + \frac{\partial}{\partial z}\left(\rho_0 f_0 \frac{\overline{v'\theta'}}{\theta_{0z}}\right)$$

➢ the EP flux divergence

We then get *diagnostic* equations for $\bar{u}_t$ and $\bar{v}^*$:

$$\rho_0 \left[\frac{\partial^2}{\partial y^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \varepsilon \frac{\partial}{\partial z}\right)\right] \bar{u}_t \; = \; (\nabla\cdot\mathbf{F})_{yy} \; - \; (\rho_0 f_0 \bar{Q}/\theta_{0z})_{yz}$$

and

$$\rho_0 \left[\frac{\partial^2}{\partial y^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \varepsilon \frac{\partial}{\partial z}\right)\right] f_0 \bar{v}^* \; = \; -[\rho_0 \varepsilon (\rho_0^{-1} \nabla\cdot\mathbf{F})_z]_z \; - \; (\rho_0 f_0 \bar{Q}/\theta_{0z})_{yz}$$

Similar to the 'omega equation'

Reduction to EP and CD

In the special case of *steady* $(\partial A/\partial t = 0)$, *non-dissipated* $(D = 0)$ waves, the GEP relation gives EP's result

$$\nabla \cdot \mathbf{F} = 0 \, .$$

Then the TEM formulation quickly leads to

$$\frac{\partial \bar{u}}{\partial t} = 0 \, ,$$

CD's *non-acceleration theorem*.

Further generalisations

- McIntyre and I originally did this for the Boussinesq primitive equations on a beta-plane, and applied it to equatorial waves and the QBO. (*JAS* 1976.)
- We also generalised it to other equation sets and spherical geometry. (*JAS* 1978.)
- At the same time, John Boyd (*JAS* **33**, 2285-2291, 1976) had similar ideas.

VOL. 33, NO. 11 JOURNAL OF THE ATMOSPHERIC SCIENCES NOVEMBER 1976

Planetary Waves in Horizontal and Vertical Shear: The Generalized Eliassen-Palm Relation and the Mean Zonal Acceleration[1]

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(Manuscript received 5 March 1976, in revised form 8 July 1976)

ABSTRACT

Using a new generalization of the Eliassen-Palm relations, we discuss the zonal-mean-flow tendency $\partial\bar{u}/\partial t$ due to waves in a stratified, rotating atmosphere, with particular attention to equatorially trapped modes. Wave transience, forcing and dissipation are taken into account in a very general way. The theory makes it possible to discuss the latitudinal (y) and vertical (z) dependence of $\partial\bar{u}/\partial t$ qualitatively and calculate it directly from an approximate knowledge of the wave structure. For equatorial modes it reveals that the y profile of $\partial\bar{u}/\partial t$ is strongly dependent on the nature of the forcing or dissipation mechanism. A by-product of the theory is a far-reaching generalization of the theorems of Charney-Drazin, Dickinson and Holton on the forcing of $\partial\bar{u}/\partial t$ by conservative linear waves.

Implications for the quasi-biennial oscillation in the equatorial stratosphere are discussed. Graphs of y profiles of $\partial\bar{u}/\partial t$ are given for the equatorial waves considered in the recent analysis of observational data by Lindzen and Tsay (1975). The y profiles of $\partial\bar{u}/\partial t$ for Rossby-gravity and inertio-gravity modes, in Lindzen and Tsay's parameter ranges, prove extremely sensitive to whether or not small amounts of mechanical dissipation are present alongside the radiative-photochemical dissipation of the waves.

The probable importance of low-frequency Rossby waves for the momentum budget of the descending easterlies is suggested.

Most other people found this paper mysterious, too!

Generalized Eliassen-Palm and Charney-Drazin Theorems for Waves on Axisymmetric Mean Flows in Compressible Atmospheres

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(Manuscript received 10 August 1976, in final form 20 September 1977)

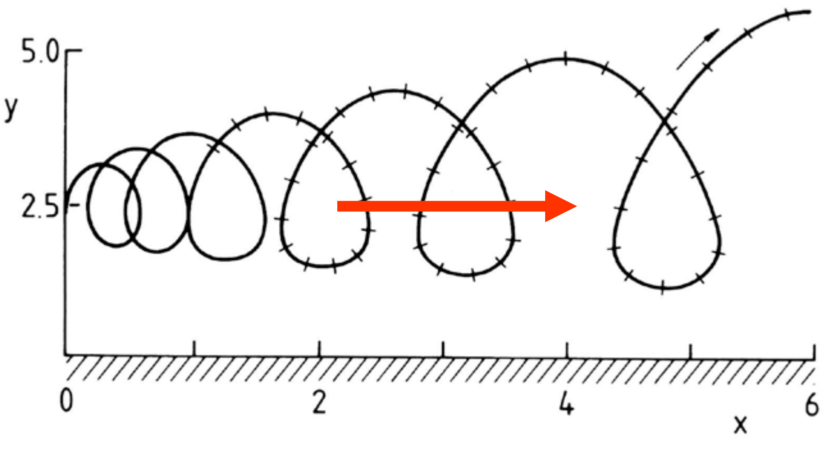
ABSTRACT

The theorems, which exhibit the role of wave dissipation, excitation and transience in the forcing of mean flow changes of second order in wave amplitude by arbitrary, small-amplitude disturbances, are obtained 1) for the primitive equations in pressure coordinates on a sphere, and 2) in a more general form (applicable for instance to nonhydrostatic disturbances in tornadoes or hurricanes) establishing that no approximations beyond axisymmetry of the mean flow are necessary. It is shown how the results reduce to those found by Boyd (1976) for the case of sinusoidal, hydrostatic waves with exponentially growing or decaying amplitude, and it is explained why the approximation used by Boyd in the thermodynamic equation is not needed. The reduction to Boyd's results entails the use of a virial theorem. This theorem amounts to a generalization of the "equipartition" law derived in an earlier paper (Andrews and McIntyre, 1976). That derivation appeared to rely on an assumption about relative phases of disturbance Fourier components; the present derivation shows that no such assumption is in fact necessary.

Lagrangian means

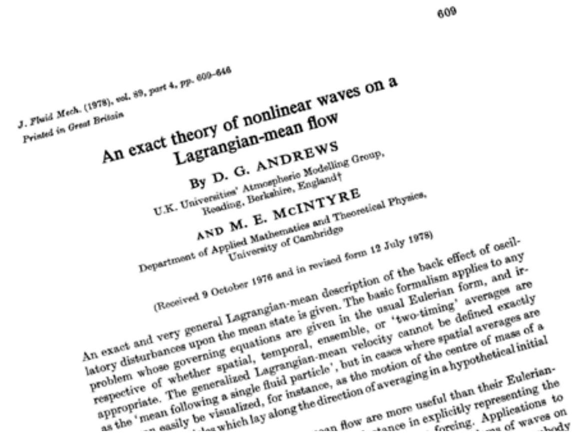
- Suggested by Bretherton in 1971, extending Stokes (1847) for water waves, and `acoustic streaming' ideas for sound waves.
- Take time-average following a fluid particle (Lagrangian mean), not at a fixed point (Eulerian mean).

Motion of a fluid particle in a growing standing wave. Eulerian mean $\overline{u}^{\mathsf{E}} = 0$, but Lagrangian mean $\overline{u}^{\mathsf{L}} \neq 0$.



Generalised Lagrangian Mean (1978)

- For finite-amplitude waves, in principle [not restricted to $O(\text{amplitude}^2)$] and includes other averages.
- Conservation laws:
  - Wave action (average over phase)
  - Pseudo-momentum (x-average)
  - Pseudo-energy (t-average)
- Finite-amplitude GLM is difficult to use in practice!



J. Fluid Mech. (1978), vol. 89, part 4, pp. 609–646
Printed in Great Britain

An exact theory of nonlinear waves on a Lagrangian-mean flow

By D. G. ANDREWS
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Department of Applied Mathematics and Theoretical Physics, University of Cambridge

(Received 9 October 1976 and in revised form 12 July 1978)

An exact and very general Lagrangian-mean description of the back effect of oscillatory disturbances upon the mean state is given. The basic formalism applies to any problem whose governing equations are given in the usual Eulerian form, and irrespective of whether spatial, temporal, ensemble, or 'two-timing' averages are appropriate. The generalized Lagrangian-mean velocity cannot be defined exactly as the 'mean following a single fluid particle', but in cases where spatial averages are ... easily be visualized, for instance, as the motion of the centre of mass of a ... which lay along the direction of averaging in a hypothetical initial ...
... flow are more useful than their Eulerian- ... in explicitly representing the ... forcing. Applications to ... of waves on

J. Fluid Mech. (1978), vol. 89, part 4, pp. 647–664
Printed in Great Britain

On wave-action and its relatives

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(Received 9 October 1976 and in revised form 12 July 1978)

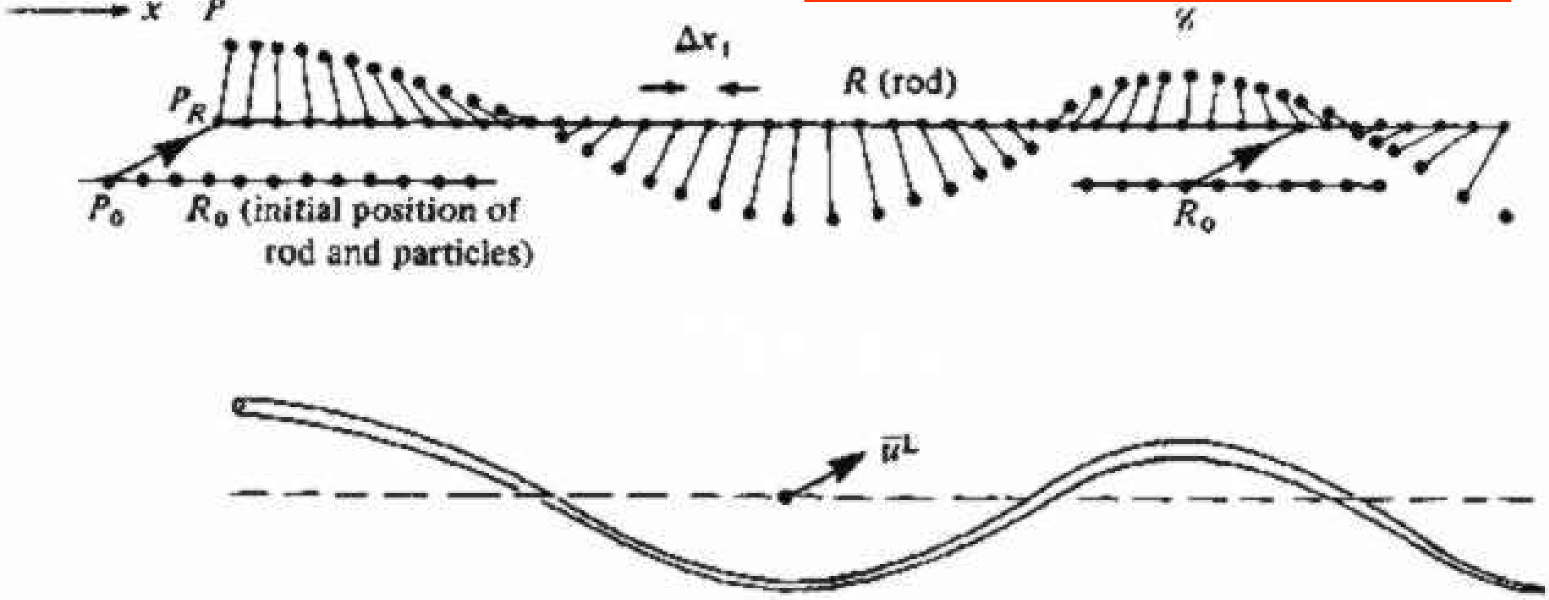
Conservable quantities measuring 'wave activity' are discussed. The equation for the most fundamental such quantity, wave-action, is derived in a simple but very general form which does not depend on the approximations of slow amplitude modulation, linearization, or conservative motion. The derivation is *elementary*, in the sense that a variational formulation of the equations of fluid motion is not used. The result depends, however, on a description of the disturbance in terms of particle displacements rather than velocities. A corollary is an elementary but general derivation of the approximate form of the wave-action equation found by Bretherton & Garrett (1968) for slowly-varying, linear waves.

The sense in which the general wave-action equation follows from the classical 'energy-momentum-tensor' formalism is discussed, bringing in the concepts of pseudomomentum and pseudoenergy, which in turn are related to special cases such as Blokhintsev's conservation law in acoustics. Wave-action, pseudomomentum and pseudoenergy are the appropriate conservable measures of wave activity when 'waves' are defined respectively as departures from ensemble-, space- and time-averaged flows.

The relationship between the wave drag on a moving boundary and the fluxes of momentum and pseudomomentum is discussed.

Interpreting particle displacements and Lagrangian mean velocity when an *x*-average is used

A&M's interpretation



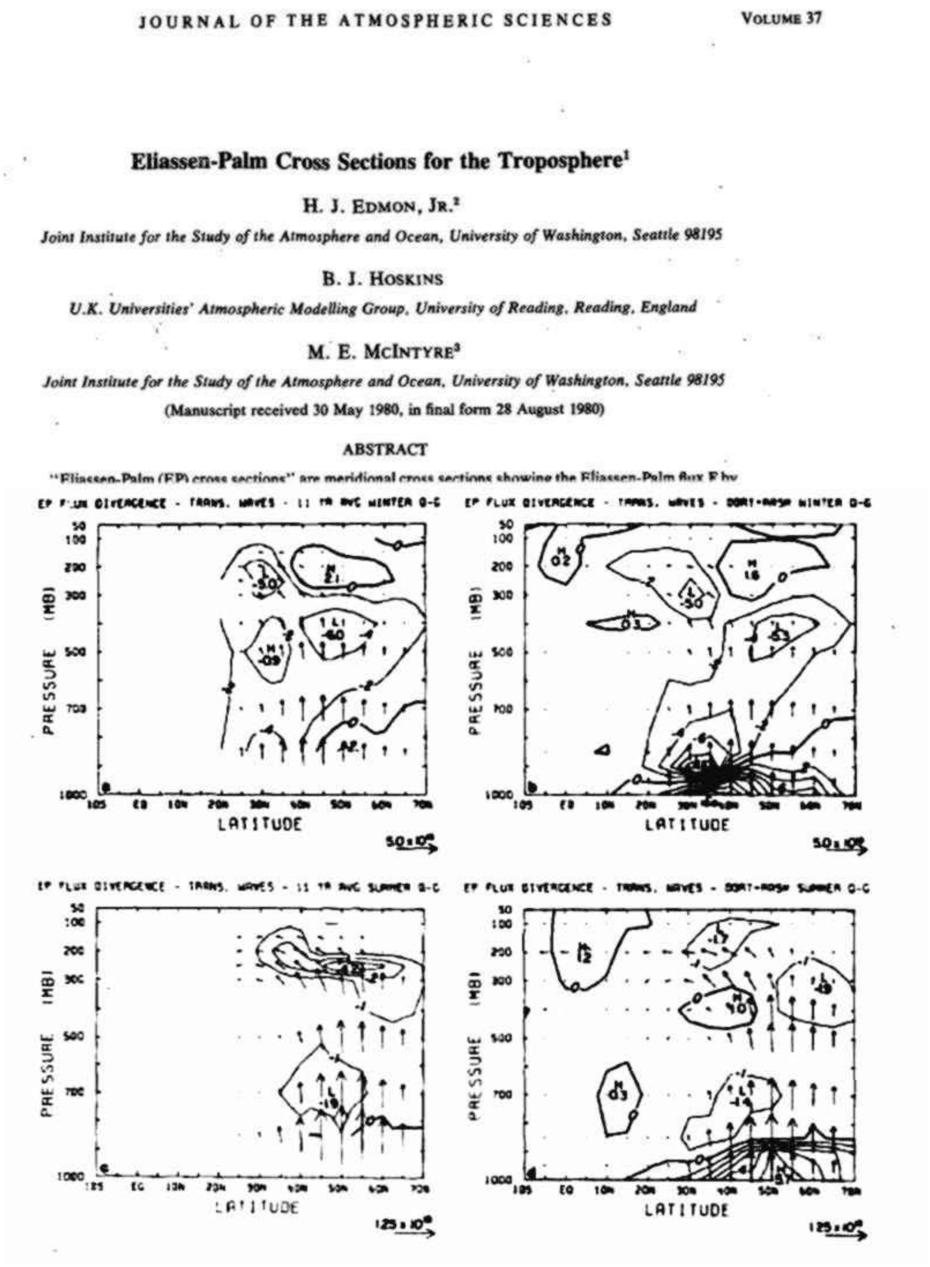
Matsuno's interpretation

This takes us up to 1978. What has happened in 30 years since?

- Many researchers (especially Japanese!) have used the transformed Eulerian mean / EP fluxes for diagnosing atmospheric waves in models and data.
- The EP flux vector ***F*** can give an idea of direction of wave propagation (generalisation of group velocity).
- Its divergence gives a force per unit mass acting on the mean flow.
- Some early examples...

EP cross-sections

Introduced by Edmon et al. (*JAS* 1980)



2600 JOURNAL OF THE ATMOSPHERIC SCIENCES VOLUME 37

Eliassen-Palm Cross Sections for the Troposphere[1]

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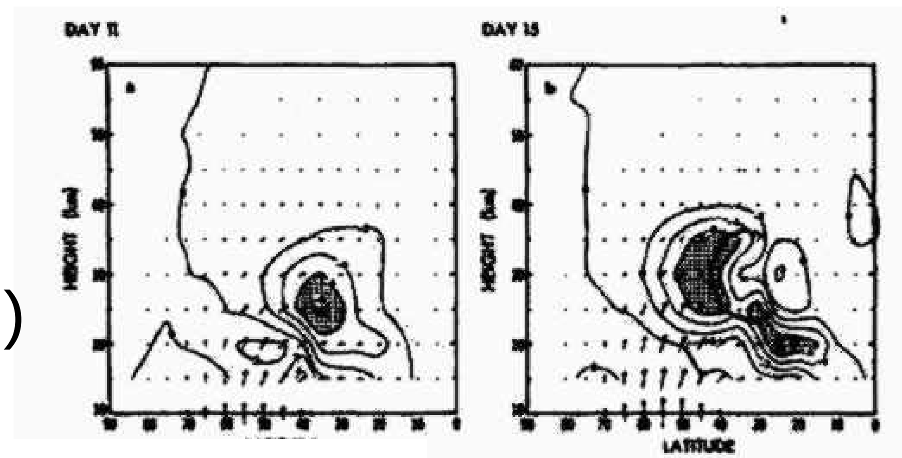
(Manuscript received 30 May 1980, in final form 28 August 1980)

ABSTRACT

"Eliassen-Palm (EP) cross sections" are meridional cross sections showing the Eliassen-Palm flux F by

Interpretation of model sudden warmings

Dunkerton et al. (*JAS* 1981)



Some Eulerian and Lagrangian Diagnostics for a Model Stratospheric Warming[1]

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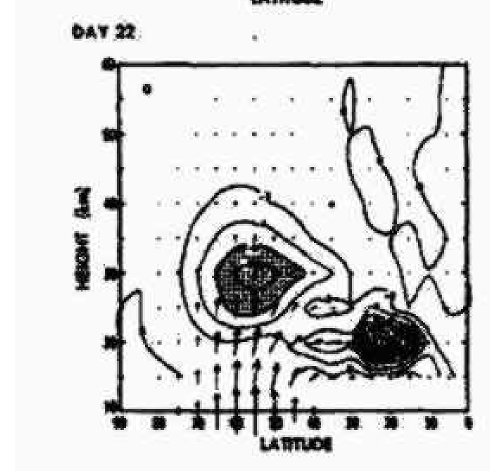
(Manuscript received 30 May 1980, in final form 11 December 1980)

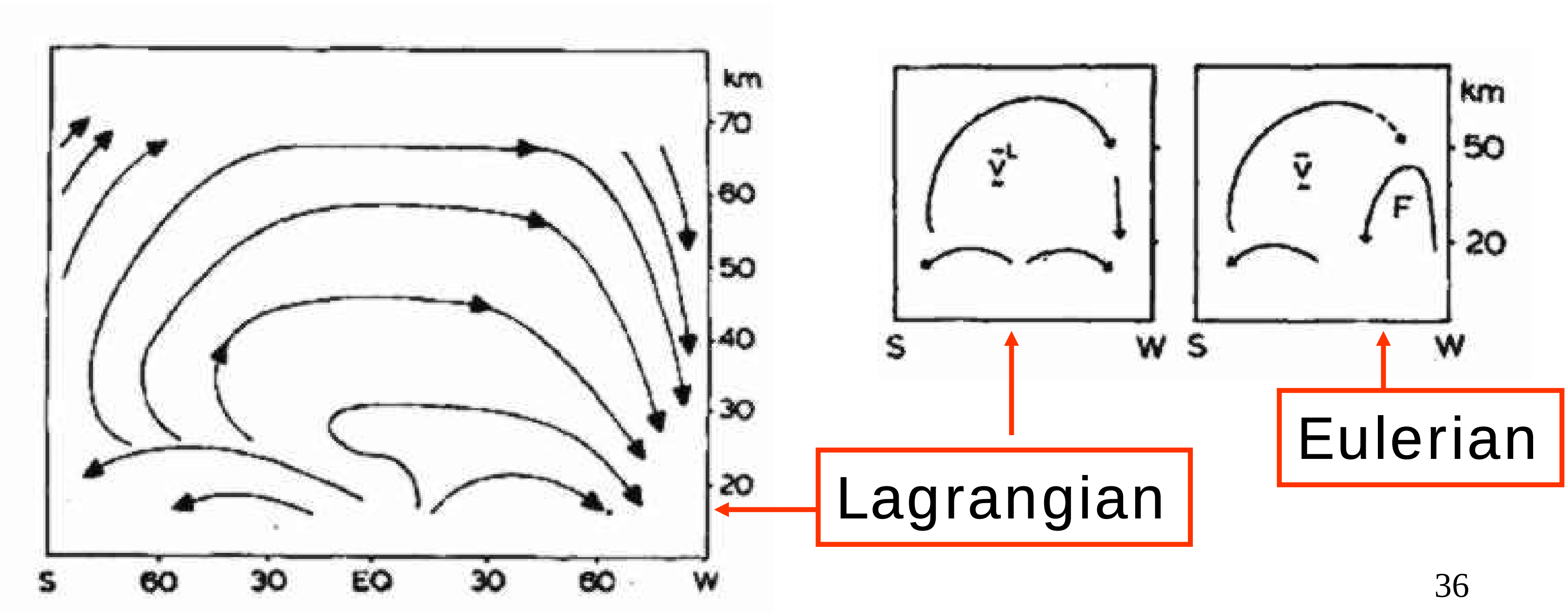
ABSTRACT

Some new diagnostics are presented for a wavenumber-2 sudden warming, simulated by a version of Holton's semi-spectral, primitive-equation model. First, Eliassen-Palm cross sections exhibiting the Eliassen-Palm (EP) planetary-wave flux together with contours of the corresponding flux divergence, are presented for selected days of the simulation. Second, a description of zonal-mean-flow evolution in the model, simpler than the conventional Eulerian-mean description and qualitatively like Lagrangian-mean descriptions in some respects, is constructed from the transformed Eulerian-mean equations presented by Andrews and McIntyre (1976). In this description the mean warming is brought about by a thermally direct "residual meridional circulation" arising as an essentially adiabatic response to a wave-induced torque about the earth's axis. The torque itself is equal to the divergence of the EP wave flux and approximately proportional to the northward flux of quasi-geostrophic potential vorticity. Third, some true Lagrangian means and related diagnostics are presented and discussed.



Tracer transport

- Variants of the TEM and GLM formalisms have been used (e.g. Dunkerton, *JAS* 1978) to diagnose wave-driven tracer transport in stratospheric models (e.g. Brewer-Dobson circulation, upper mesospheric circulation).



An unusual application: orthogonality of modes in shear flow

- Held, *JAS* 1985: linear modes in shear are not orthogonal in `energy' sense, i.e. for 2 modes the total energy $\neq$ sum of energies of separate modes.
- However, they are orthogonal in the pseudo-energy or pseudo-momentum sense.

$$\text{Inner product} = \int_0^L \frac{\partial \bar{q}}{\partial y} \eta_1^* \eta_2 \, dy$$

More recently...

- There have been many other applications of the theory.
- I have not done much work in this area for many years, and I am not familiar with them all!
- However, recently I have been collaborating with researchers in the UK Met Office, to help set up EP diagnostics suitable for their ‘non-hydrostatic’ GCM.

- 2 weeks ago I was asked to review yet another paper on a variant of the Generalised Lagrangian Mean...!

Limitations of the approach

- EP diagnostics may not work well for large-amplitude disturbances (e.g. breaking Rossby waves in the stratosphere, baroclinic waves in the troposphere).
  - ‘Wave, mean’ separation may not be appropriate then.
- Potential vorticity diagnostics may be more useful in these cases.

Final point

- This theory has shown that there is no unique way of defining ‘wave’ and ‘mean’ quantities.
- Formulations such as the TEM and GLM may be better than the Eulerian mean for interpreting some processes.
- But the Eulerian mean may still be the best for other purposes.

The end