Earth-like circulation
and
Venus-like circulation

*Hiroki Kashimura (ISAS/JAXA)

(*Formerly known as Hiroki Yamamoto)

Earth-like circulation
and
Venus-like circulation

in an Idealized Axisymmetric model

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1 Introduction

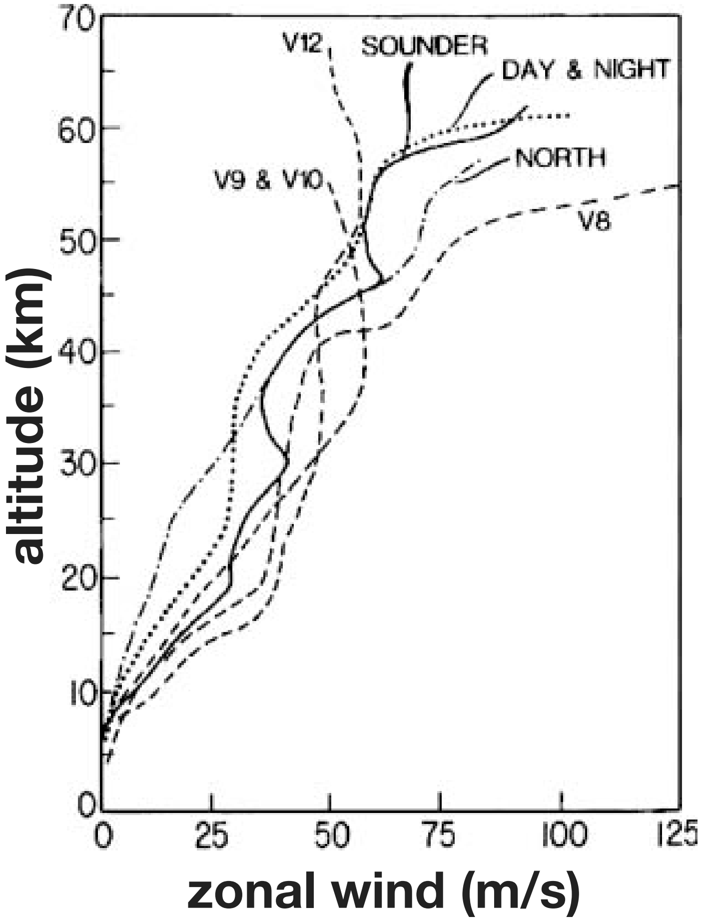
1 1 Observation | Venus

- Venus’ size and mass are similar to Earth’s.
  - radius ~ 6052 km, gravity ~ 8.9 m/s^2

- Venus rotates very slow.
  - rotation period ~ 243 (Earth) days

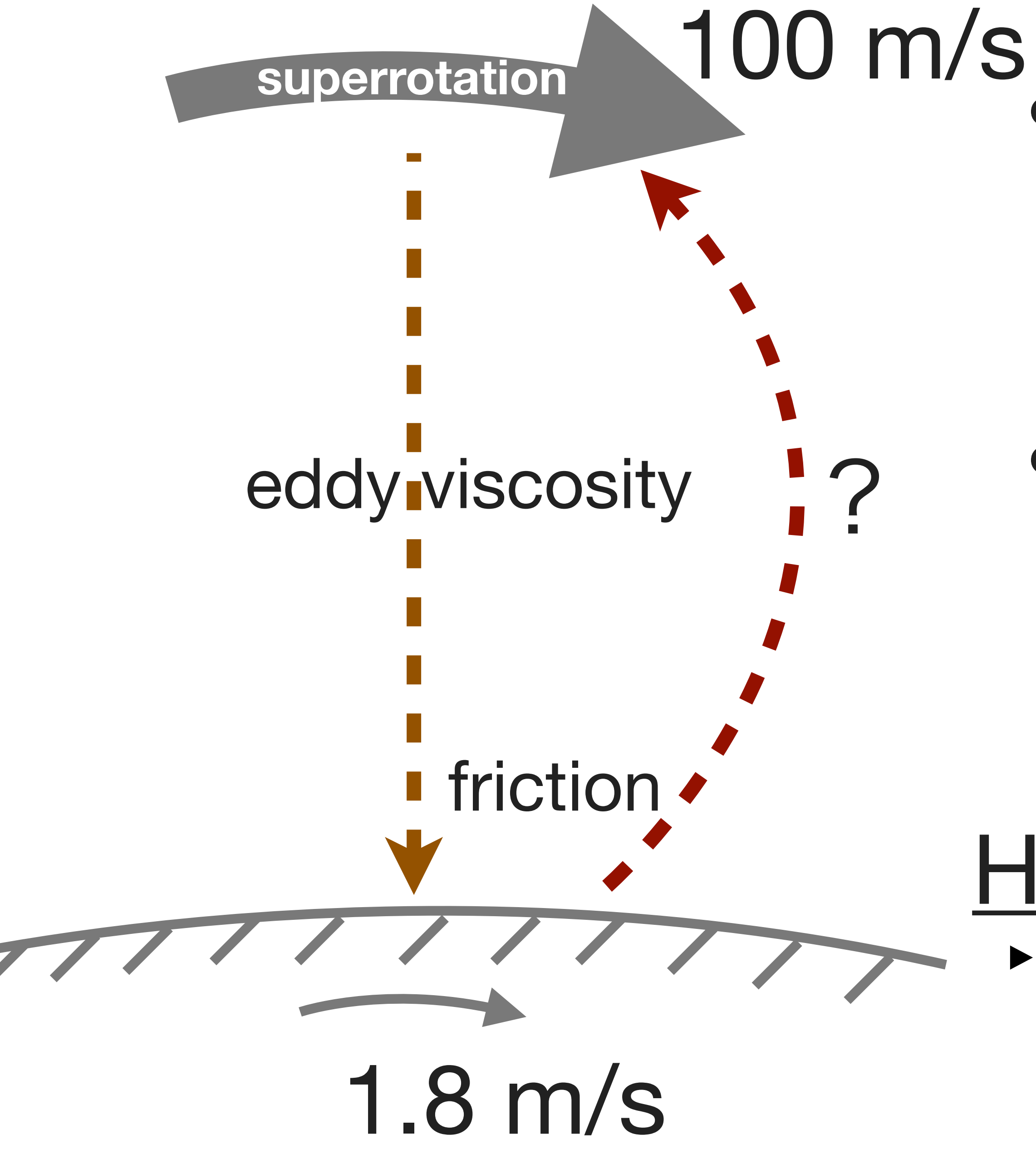
- Zonal wind in the Venus atmosphere reaches ~ 100 m/s.
  - This is **60 times faster** than the planetary rotation at the equator!

Superrotaiton



Schubert (1983)

1-2 Mystery of superrotation



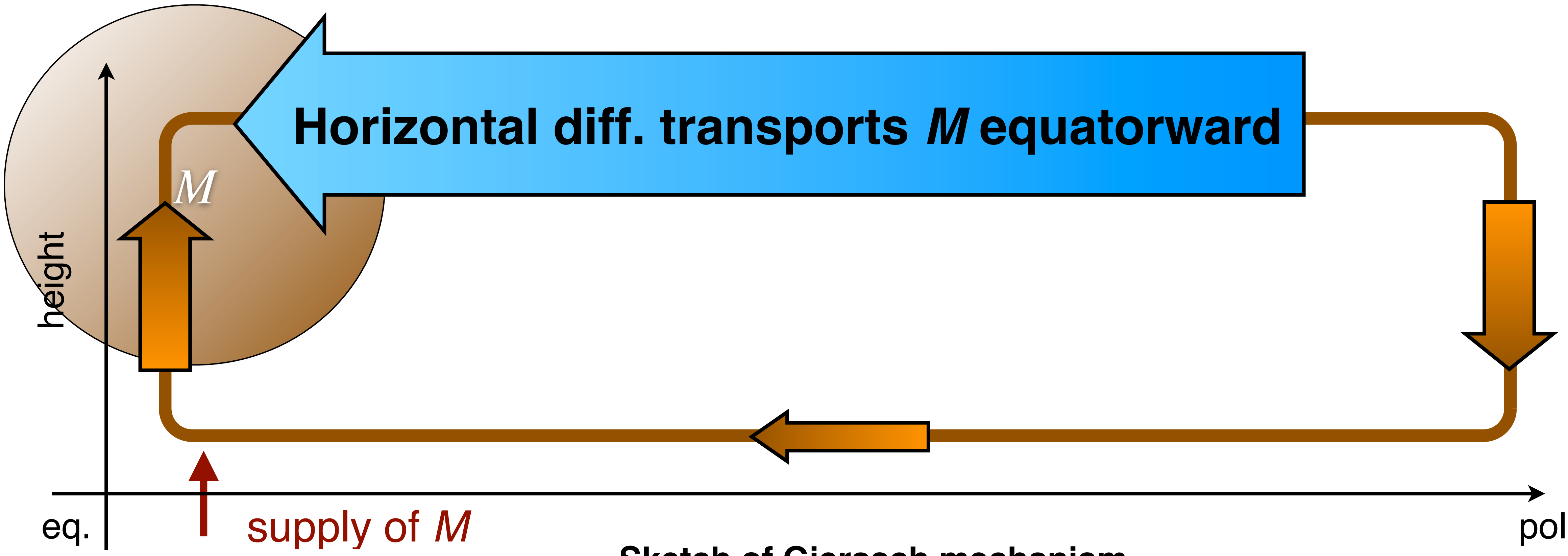
- Eddy viscosity and surface friction slow down the atmos.
- Angular momentum must be pumped up to the atmosphere by some mechanism.

Hypotheses

- dayside-nightside circulation (e.g., Schubert & Whitehead, 1969)
- gravity waves (e.g., Fels & Linzen, 1974)
- meridional circulation (e.g., Gierasch, 1975)

1 3 Gierasch (1975) mechanism

- Gierasch (1975) assumed
  - symmetries about the rotation axis and the equator
  - Hadley cell expanding from the equator to the pole
  - infinitely large *horizontal diffusion*
    - ➡ always solid body rotation = constant angular vel.

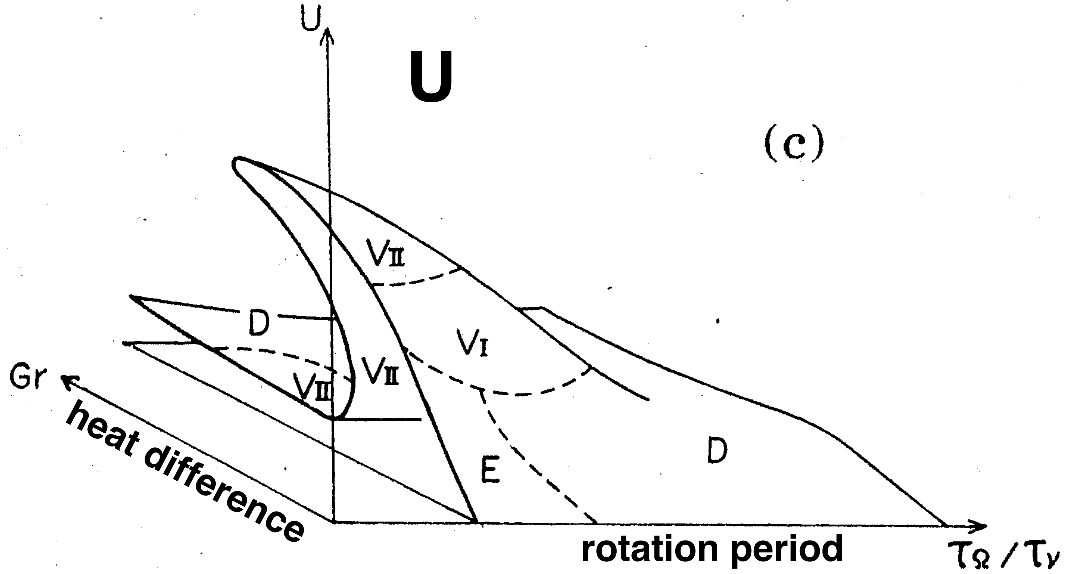


Sketch of Gierasch mechanism

1 4 Gierasch mechanism

- Gierasch stated that, for the mechanism to work,
  - relaxation time of the horizontal diffusion must be
    - much shorter than that of vertical diffusion
    - and turnover time of meridional circulation.

- Matsuda (1980, 1982) explored
  - parameter dependency of U,
  - dominant moment balance,
  - multiple equilibrium solutions,



for both infinite and finite hor. diff. cases,
using a highly truncated low-order spectral model.

1 5 In my study

We theoretically and numerically explore the strength of the superrotation maintained by the Gierasch mechanism using an idealized system.

In Matsuda (1980, 1982)		<u>In this study, we</u>
• advection of potential temperature was ignored,	➡	treat the meridional temp. difference as an internal variable,
• math was complicated because of mode equations,	➡	develop a theoretical model expressed by algebraic equations for superrotation strength
• theoretical results were not verified by numerical experiments with high orders.	➡	verify the theoretical solution by numerical experiments

2 1 Basic equations

- primitive equations
- dry Boussinesq fluid
- Newtonian heating and cooling
- axisymmetric with strong horizontal diffusion of momentum

$$\frac{\partial u}{\partial t} + \frac{v}{a}\frac{\partial u}{\partial \phi} + w\frac{\partial u}{\partial z} - \frac{uv\tan\phi}{a} - 2\Omega v\sin\phi = \nu_H D_H(u) + \nu_V\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + \frac{v}{a}\frac{\partial v}{\partial \phi} + w\frac{\partial v}{\partial z} + \frac{u^2\tan\phi}{a} + 2\Omega u\sin\phi = -\frac{1}{a}\frac{\partial \Phi}{\partial \phi} + \nu_H D_H(v) + \nu_V\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial \Phi}{\partial z} = g\frac{\theta - \Theta_0}{\Theta_0}$$

$$\frac{\partial \theta}{\partial t} + \frac{v}{a}\frac{\partial \theta}{\partial \phi} + w\frac{\partial \theta}{\partial z} = -\frac{\theta - \theta_e}{\tau} + \kappa_V\frac{\partial^2 \theta}{\partial z^2} \qquad \theta_e \equiv \Theta_0\left[1 - \Delta_H\left(\sin^2\phi - \frac{1}{3}\right)\right]$$

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(v\cos\phi) + \frac{\partial w}{\partial z} = 0$$

2 2 Boundary conditions

- Top: free-slip, no mass or heat flux,

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = w = \frac{\partial \theta}{\partial z} = 0 \quad (\text{ top } : z = H)$$

- Bottom: no-slip, no mass or heat flux

$$u = v = w = \frac{\partial \theta}{\partial z} = 0 \quad (\text{bottom}:\ z = 0)$$

- Equator and pole: no mass, momentum, or heat flux

$$v = \frac{\partial u}{\partial \phi} = \frac{\partial w}{\partial \phi} = \frac{\partial \theta}{\partial \phi} = 0 \quad \left(\text{eq.} : \phi = 0, \text{pole}: \phi = \frac{\pi}{2} \right)$$

3 1 How to develop a theoretical model

Basic equations are nonlinear PDEs, which we cannot solve analytically.

We assume
steady state: $\partial/\partial t = 0$
spacial structure of some variables

spacial integral, relation of scales

Theoretical model which is a set of algebraic equations for four nondimensional variables

3 2 How to develop a theoretical model

Theoretical model which is a set of algebraic equations for four nondimensional variables

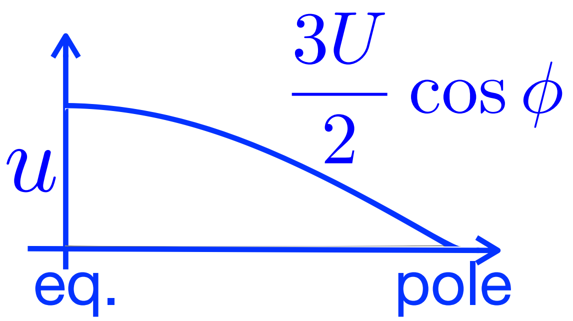
superrotation strength $S \equiv \frac{U}{a\Omega}$ $R_{vB} \equiv \frac{V_B}{a\Omega}$ $R_{vT} \equiv \frac{V_T}{a\Omega}$ $\beta \equiv \frac{\Delta\Theta}{\Theta_0 \Delta_H}$

meridional mean zonal wind at the top

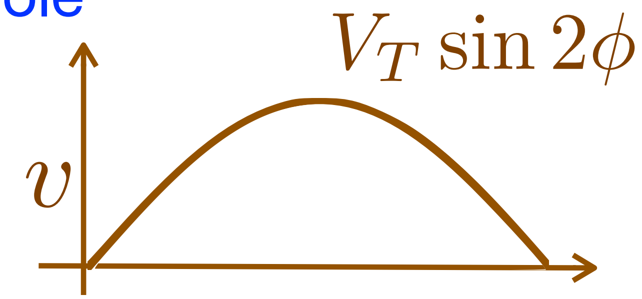
- low-order system
- enable to solve analytically
- easy to see the parameter dependency

3 3 Assumption: spacial structure of variables

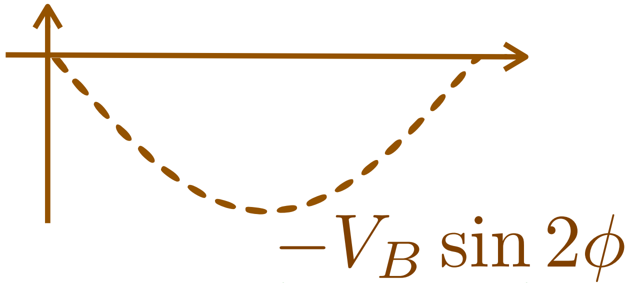
- zonal wind at the top:



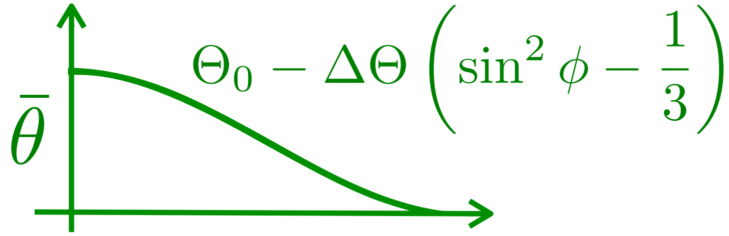
- meridional wind at the top:



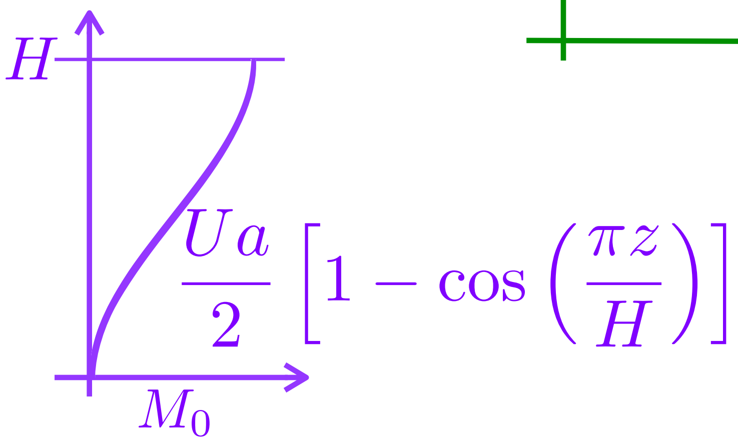
- meridional wind above the surface:



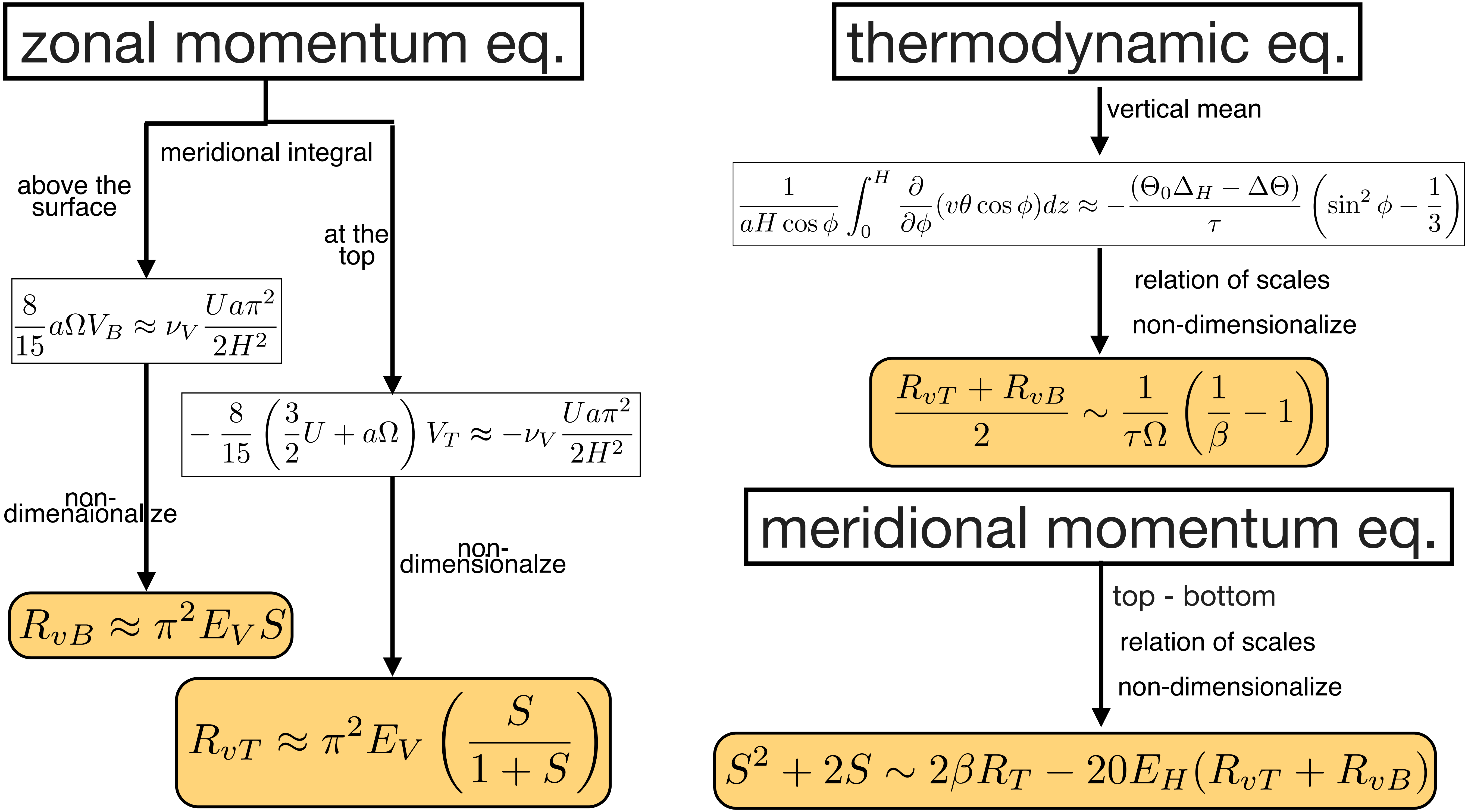
- vertical mean of potential temp.:



- meridional integral of angular momentum:



3 4 Development of the theoretical model



zonal momentum eq.

meridional integral

above the surface

$$\frac{8}{15}a\Omega V_B \approx \nu_V \frac{U a \pi^2}{2H^2}$$

non-dimensionalize

$$R_{vB} \approx \pi^2 E_V S$$

at the top

$$-\frac{8}{15}\left(\frac{3}{2}U + a\Omega\right)V_T \approx -\nu_V \frac{U a \pi^2}{2H^2}$$

non-dimensionalze

$$R_{vT} \approx \pi^2 E_V \left(\frac{S}{1+S}\right)$$

thermodynamic eq.

vertical mean

$$\frac{1}{aH\cos\phi}\int_0^H \frac{\partial}{\partial\phi}(v\theta\cos\phi)dz \approx -\frac{(\Theta_0\Delta_H - \Delta\Theta)}{\tau}\left(\sin^2\phi - \frac{1}{3}\right)$$

relation of scales

non-dimensionalize

$$\frac{R_{vT} + R_{vB}}{2} \sim \frac{1}{\tau\Omega}\left(\frac{1}{\beta} - 1\right)$$

meridional momentum eq.

top - bottom

relation of scales

non-dimensionalize

$$S^2 + 2S \sim 2\beta R_T - 20E_H(R_{vT} + R_{vB})$$

E_V: vertical Ekman num., $\equiv \nu_V/(H^2\Omega)$

E_H: horizonta Ekman num., $\equiv \nu_H/(a^2\Omega)$

R_T: external thermal Rossby num. $\equiv gH\Delta_H/(a^2\Omega^2)$

3_5 Algebra

algebraic equations for $\boldsymbol{S, R_{vB}, R_{vT}, \beta}$

$$R_{vB} = \pi^2 E_V S$$

$$R_{vT} = \pi^2 E_V \left(\frac{S}{1+S}\right)$$

$$\frac{R_{vT}+R_{vB}}{2} = \frac{1}{\tau\Omega}\left(\frac{1}{\beta}-1\right)$$

$$S^2 + 2S = 2\beta R_T - 20 E_H (R_{vT}+R_{vB})$$

◯ external parameters

eliminate R_{vB}, R_{vT}, β

Quintic equation for S

$$\left[S^2 + 2S + BS\left(\frac{2+S}{1+S}\right)\right]\left[\frac{AS}{2}\left(\frac{2+S}{1+S}\right)+1\right] = 2R_T$$

$$A \equiv \pi^2 \tau \Omega E_V \qquad B \equiv 20\pi^2 E_H E_V$$

③₆ Quintic equation for S

$$\left[S^2+2S+BS\left(\frac{2+S}{1+S}\right)\right]\left[\frac{AS}{2}\left(\frac{2+S}{1+S}\right)+1\right]=2R_T$$

This eq. has only one positive solution.
The positive solution estimates the superrotation strength.

It depends only on three external parameters

$$A\equiv\pi^2\tau\Omega E_V=\pi^2\frac{\tau}{H^2/\nu_V}$$

: the ratio of the radiative relaxation time to the timescale for vertical eddy diffusion of momentum

$$B\equiv 20\pi^2E_HE_V=5\left(\frac{2\pi/\Omega}{\sqrt{(a^2/\nu_H)(H^2/\nu_V)}}\right)^2$$

: the ratio of the rotation period to the geometric mean of the timescales for horizontal and vertical eddy diffusion

$$R_T\equiv\frac{gH\Delta_H}{a^2\Omega^2}$$

: the external thermal Rossby number

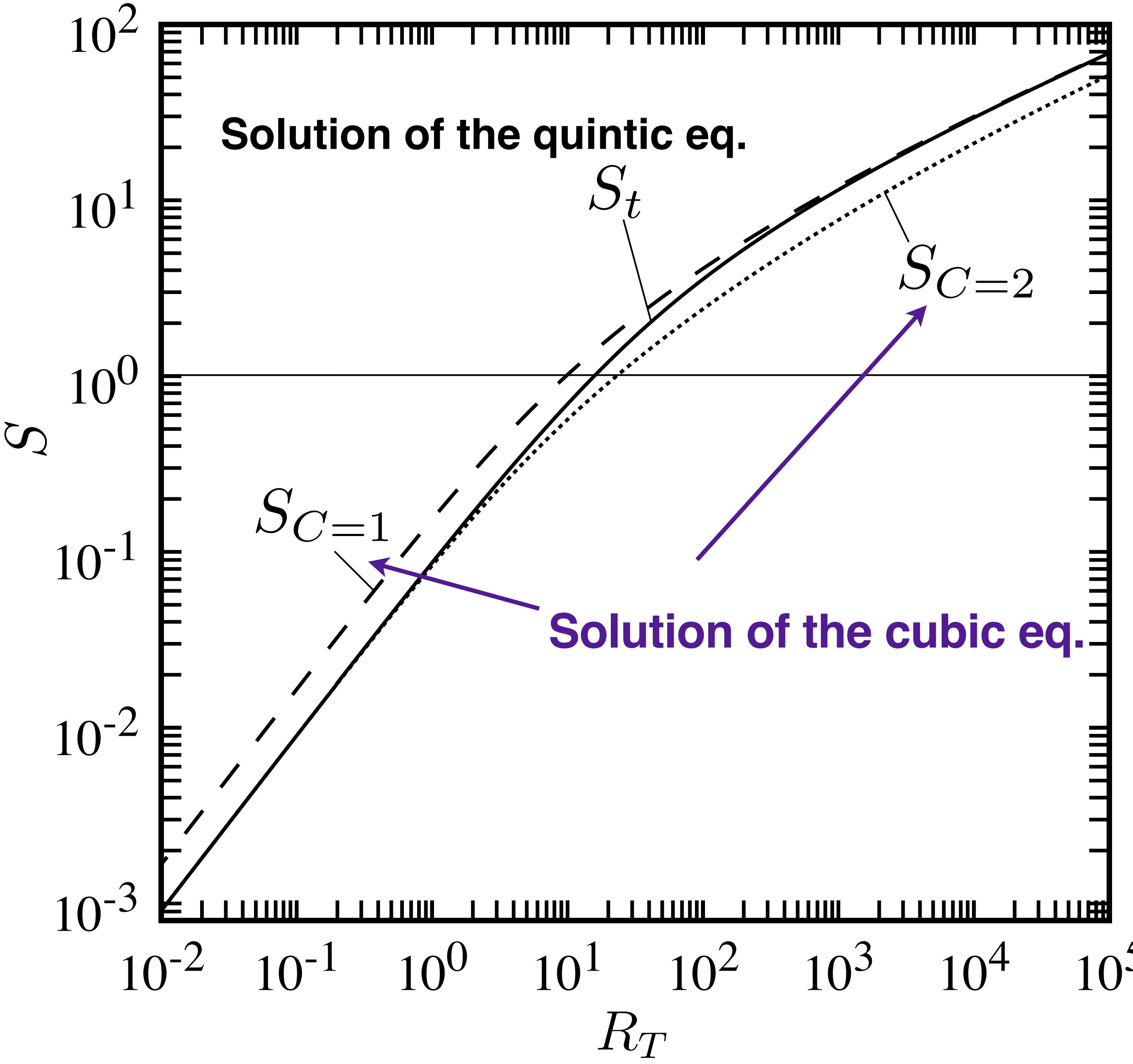
3 7 Approximation of the quintic eq.

$$\left[S^2 + 2S + BS\left(\frac{2+S}{1+S}\right)\right]\left[\right.$$

$$1 < \left(\frac{2 -}{1 -}\right.$$

$$\left(S^2 + 2S + BCS\right)\left(\frac{ACS}{2} + 1\right) \approx 2R$$

Cubic equation for S Solvab



$$\boxed{\frac{AC}{2}S^3 + \left(1 + AC + \frac{ABC^2}{2}\right)S^2 + (2 + BC)S \approx 2R_T}$$

3 8 Further approximation

From the cubic eq., S can be approximated as...

$$\boxed{\frac{AC}{2}S^3+\left(1+AC+\frac{ABC^2}{2}\right)S^2+(2+BC)S\approx 2R_T}$$

$S \approx$

R_{T1} $\quad R_T \quad$ R_{T2}

						R_{T1}	R_{T2}
(a)	$B \ll 1$	$A \ll 1$	R_T	$\sqrt{2R_T}$	$\left(\frac{4R_T}{AC}\right)^{\frac{1}{3}}$	2	$\frac{2}{A^2C^2}$
(b)		$A \gg 1$		$\sqrt{\frac{2R_T}{AC}}$		$\frac{2}{AC}$	$2AC$
(c)	$B \gg 1$	$AB \ll 1$	$\frac{2R_T}{BC}$	$\sqrt{2R_T}$		$\frac{B^2C^2}{2}$	$\frac{2}{A^2C^2}$
(d)		$AB \ll 1$		$\sqrt{\frac{2R_T}{ABC^2}}$		$\frac{B}{A}$	$\frac{AB^3C^4}{4}$

(a)	$B \ll 1$	$A \ll 1$	**G1** R_T	$\sqrt{2R_T}$ **C1**	$\left(\frac{4R_T}{AC}\right)^{\frac{1}{3}}$ **C0**	2	$\frac{2}{A^2C^2}$
(b)		$A \gg 1$		$\sqrt{\frac{2R_T}{AC}}$ **G0**		$\frac{2}{AC}$	$2AC$
(c)	$B \gg 1$	$AB \ll 1$	**H1** $\frac{2R_T}{BC}$	$\sqrt{2R_T}$ **C1**		$\frac{B^2C^2}{2}$	$\frac{2}{A^2C^2}$
(d)		$AB \ll 1$		$\sqrt{\frac{2R_T}{ABC^2}}$ **H0**		$\frac{B}{A}$	$\frac{AB^3C^4}{4}$

returning to the meridional eq. style

$$S^2 + 2S + BCS \approx 2R_T\beta = \left(\frac{ACS}{2} + 1\right)^{-1} = \frac{\Delta\Theta}{\Theta_0\Delta_H}$$

1. 2. 3.

which is dominant?

1. 2.

which is dominant?

1. cyclostrophic balance [C]
2. geostrophic balance [G]
3. horizontal diff. balance [H]

1. thermal advection is dominant [0]
2. thermal advection is ignorable [1]

0 Numerical experiments

4.1 Numerical experiments

- Time-integration was performed from motionless state.
- We obtain steady or statistically steady numerical solutions.
- External parameters are...

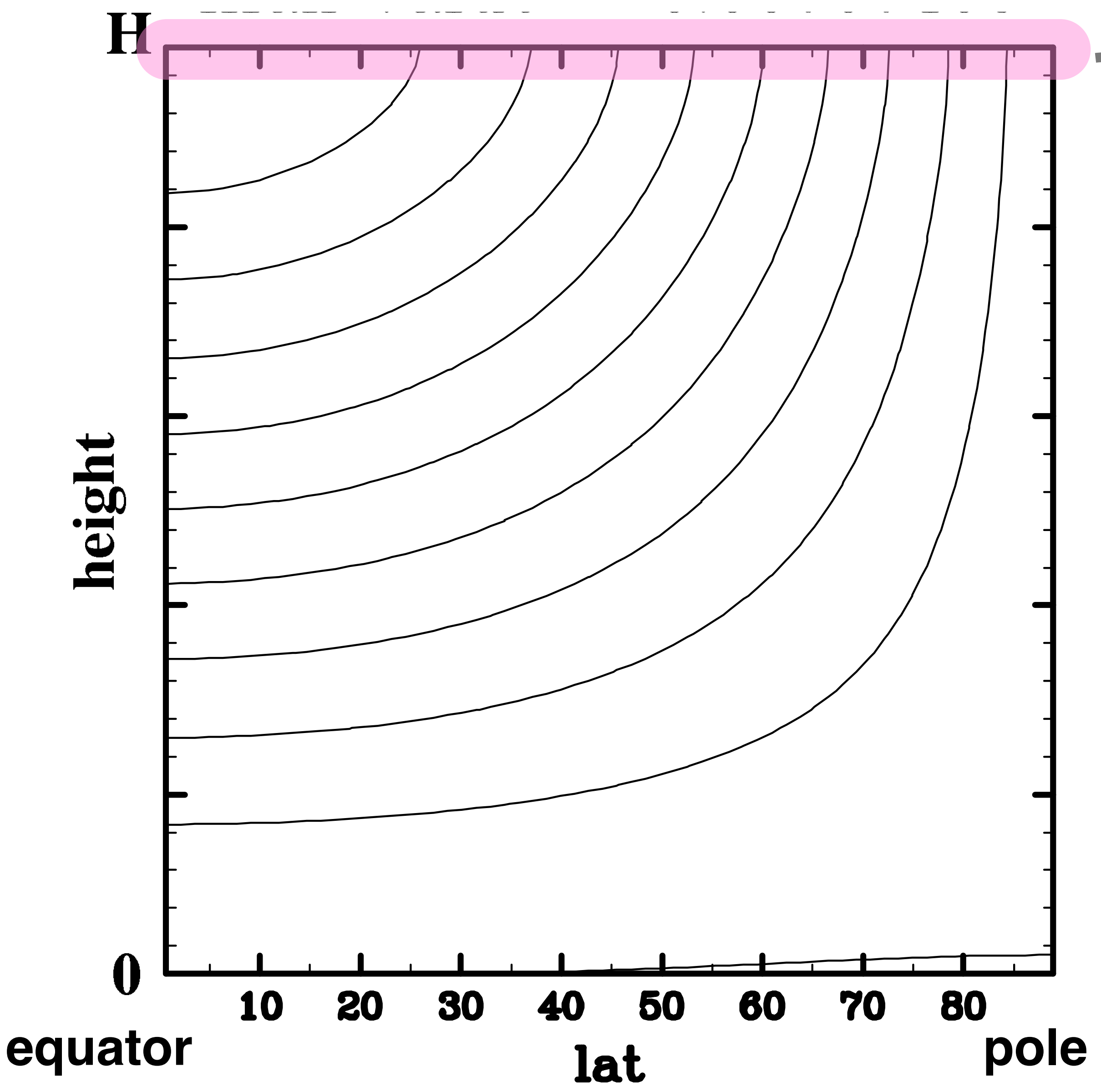
	A	B	E_V	$\tau\Omega$	E_H
(a)	$\pi^2 \times 10^{-2}$	$2\pi^2 \times 10^{-2}$	10^{-3}	10	1
(b)	π^2	$2\pi^2 \times 10^{-2}$	10^{-3}	10^3	1
(c)	$\pi^2 \times 10^{-3}$	$2\pi^2$	10^{-3}	1	10^2
(d)	$\pi^2 \times 10^{-1}$	$2\pi^2$	10^{-3}	10^2	10^2

For each combination,

$$R_T = 10^n \,(n = -2, -1, 0, ..., 5)$$ is calculated.

$A = \pi^2 \tau\Omega E_V, B = 20\pi^2 E_H E_V$

4 2 Numerical superrotation strength



Meridional mean zonal wind at the top

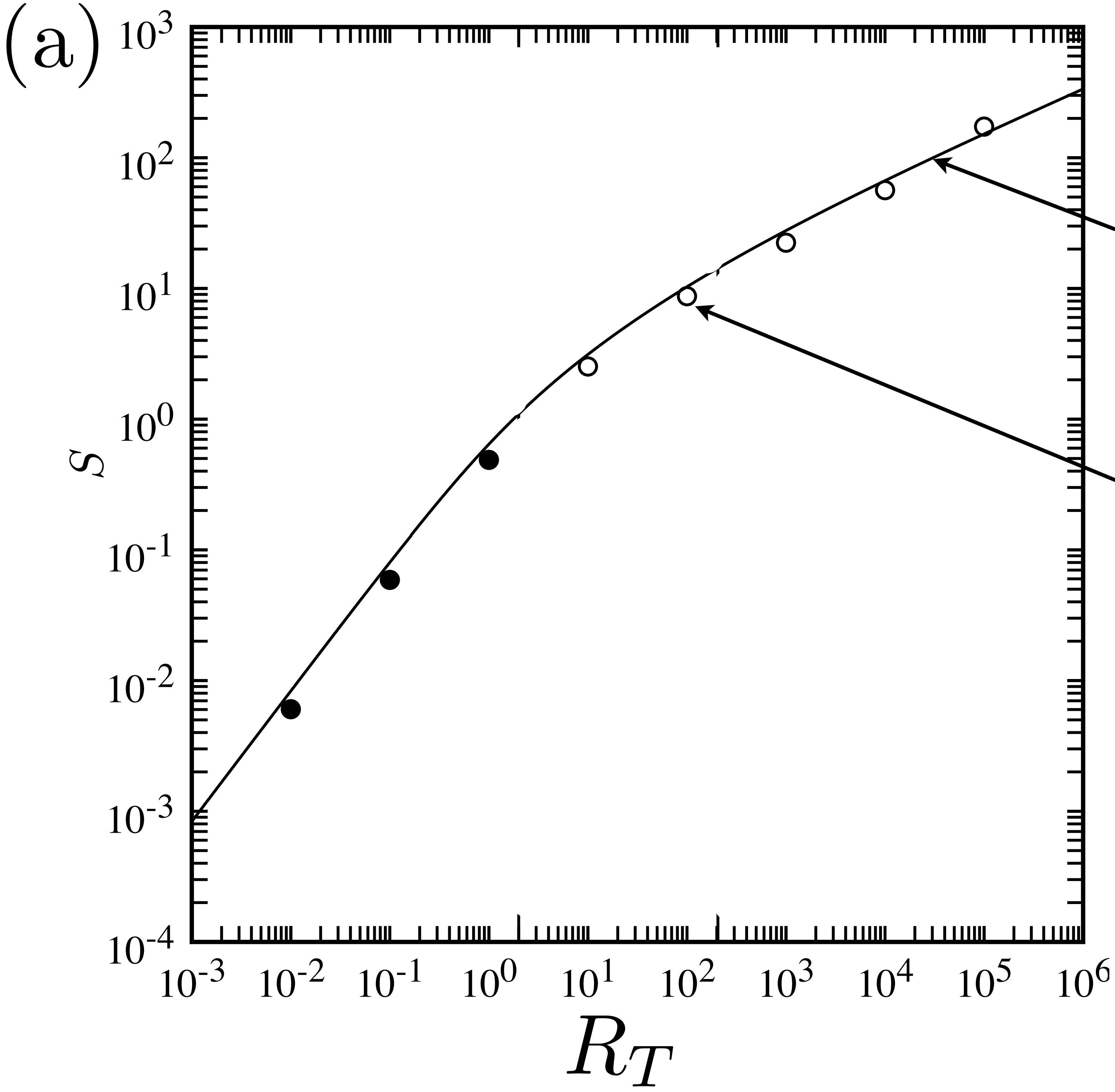
$$S_n = \frac{U}{a\Omega}$$

numerical superrotation strength

Nondimensional zonal wind field

$(u/a\Omega)$

4 3 Theoretical vs. Numerical



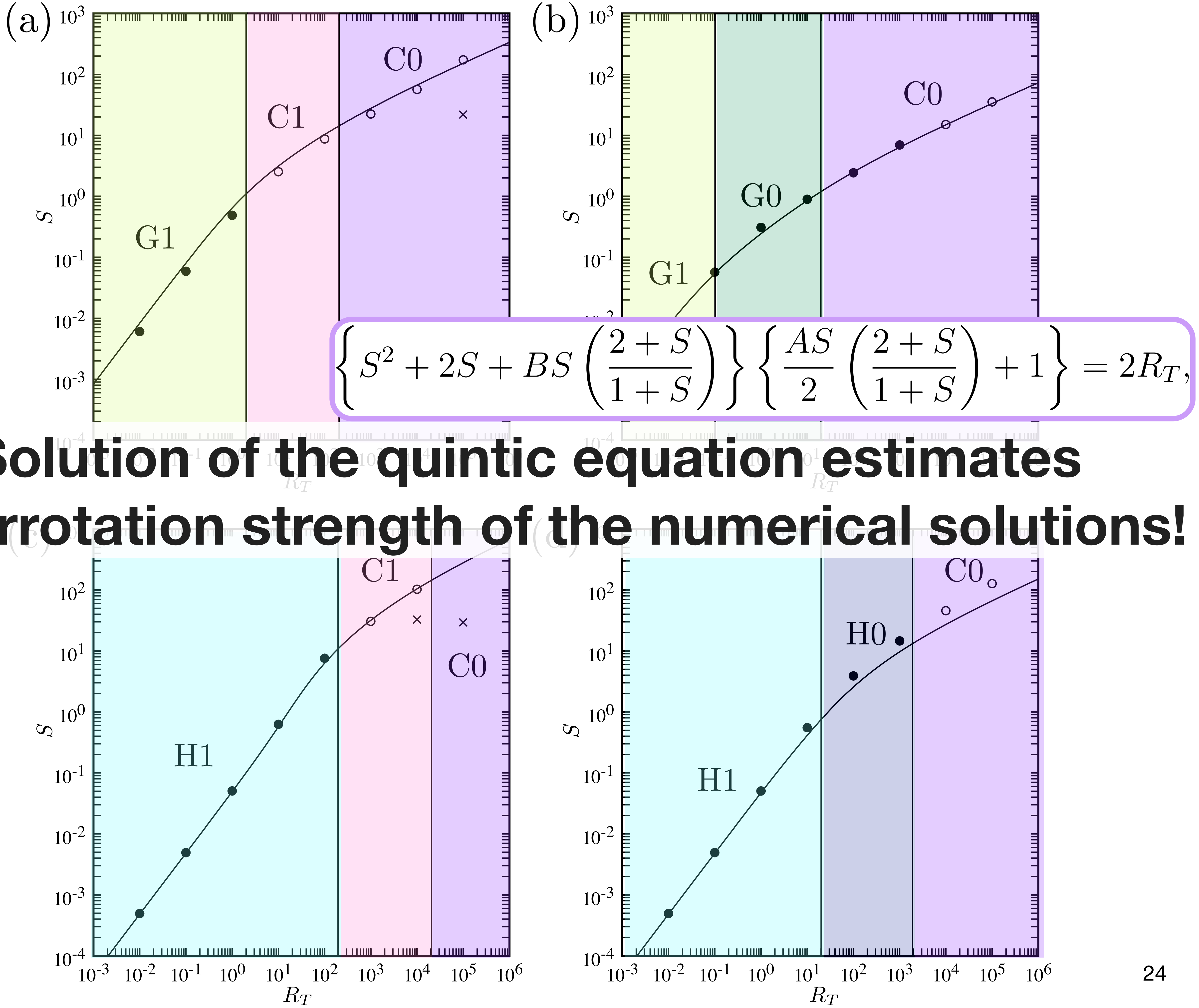
solid curve: solution of the quintic equation

$$\left\{S^2+2S+BS\left(\frac{2+S}{1+S}\right)\right\}\left\{\frac{AS}{2}\left(\frac{2+S}{1+S}\right)+1\right\}=2R_T,$$

S_n **: numerical**

● Steady state

○ Statistically-steady state



$$\left\{S^2+2S+BS\left(\frac{2+S}{1+S}\right)\right\}\left\{\frac{AS}{2}\left(\frac{2+S}{1+S}\right)+1\right\}=2R_T,$$

Solution of the quintic equation estimates superrotation strength of the numerical solutions!

5 To Earth-like circulation

5 1 Basic equations

- primitive equations
- dry Boussinesq fluid
- Newtonian heating and cooling
- axisymmetric ~~with strong horizontal diffusion~~

without horizontal diffusion

$$\frac{\partial u}{\partial t} + \frac{v}{a}\frac{\partial u}{\partial \phi} + w\frac{\partial u}{\partial z} - \frac{uv\tan\phi}{a} - 2\Omega v \sin\phi = \cancel{\nu_H D_H(u)} + \nu_V \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + \frac{v}{a}\frac{\partial v}{\partial \phi} + w\frac{\partial v}{\partial z} + \frac{u^2\tan\phi}{a} + 2\Omega u \sin\phi = -\frac{1}{a}\frac{\partial \Phi}{\partial \phi} + \cancel{\nu_H D_H(v)} + \nu_V \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial \Phi}{\partial z} = g\frac{\theta - \Theta_0}{\Theta_0}$$

$$\frac{\partial \theta}{\partial t} + \frac{v}{a}\frac{\partial \theta}{\partial \phi} + w\frac{\partial \theta}{\partial z} = -\frac{\theta - \theta_e}{\tau} + \kappa_V \frac{\partial^2 \theta}{\partial z^2} \qquad \theta_e \equiv \Theta_0\left[1 - \Delta_H\left(\sin^2\phi - \frac{1}{3}\right)\right]$$

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(v\cos\phi) + \frac{\partial w}{\partial z} = 0$$

5 2 Held and Hou (1980) model

- Axisymmetric theoretical model of the Hadley circulation of the Earth.

Theory
Width of Hadley circulation

$$\phi_H = \left(\frac{5}{3}R_T\right)^{\frac{1}{2}}$$

Hou (1984)

$$\sin\phi_H = \begin{cases} \left(\frac{5}{3}R_T\right)^{\frac{1}{2}} & (R_T \ll 1) \\ 1 - \frac{3}{8R_T} & (R_T \gg 1) \end{cases}$$

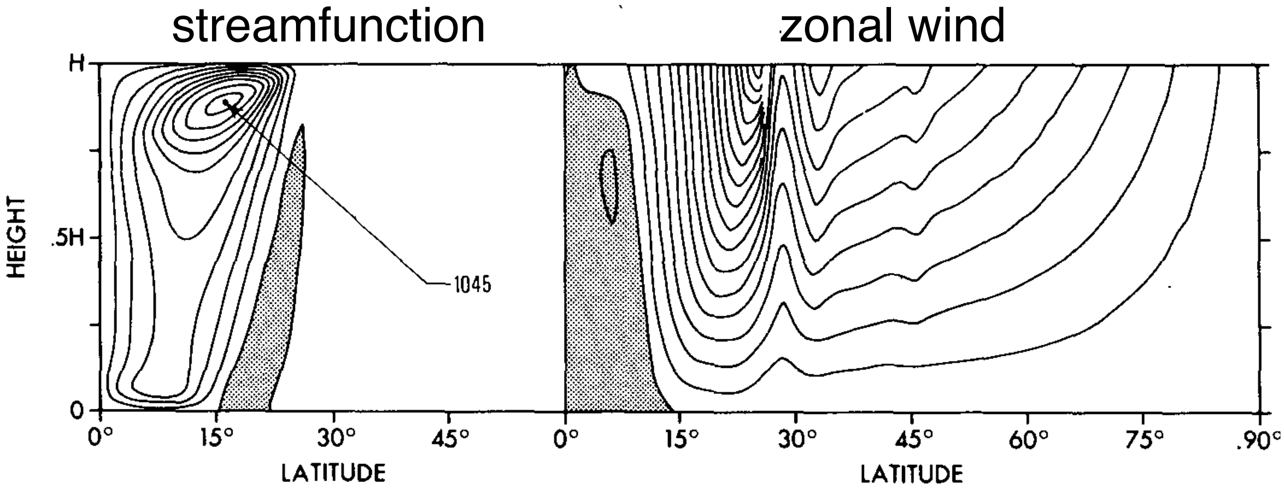
Zonal wind at the top

angular momentum conservation

$$u_{\mathrm{HH}} \equiv \begin{cases} u_M = a\Omega\frac{\sin^2\phi}{\cos\phi} & (0 \le \phi < \phi_H) \\ u_E = a\Omega\left[\left(1 + \frac{2R_T z}{H}\right)^{\frac{1}{2}} - 1\right]\cos\phi & (\phi_H \le \phi \le \pi/2) \end{cases}$$

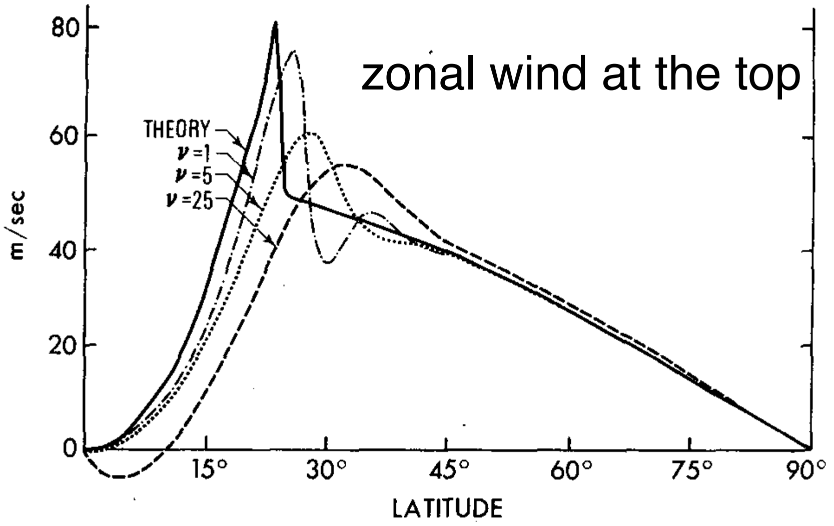
Thermal wind with respect to θ_e

Numerical solution



streamfunction

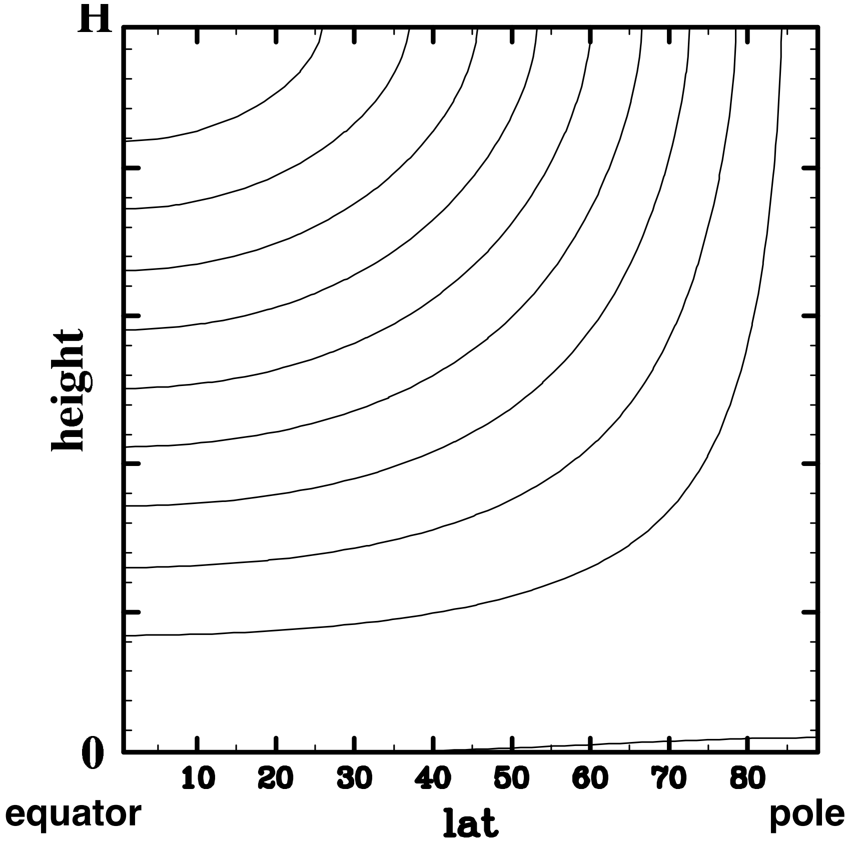
zonal wind



zonal wind at the top

Held and Hou (1980)

5 3 How solution changes?

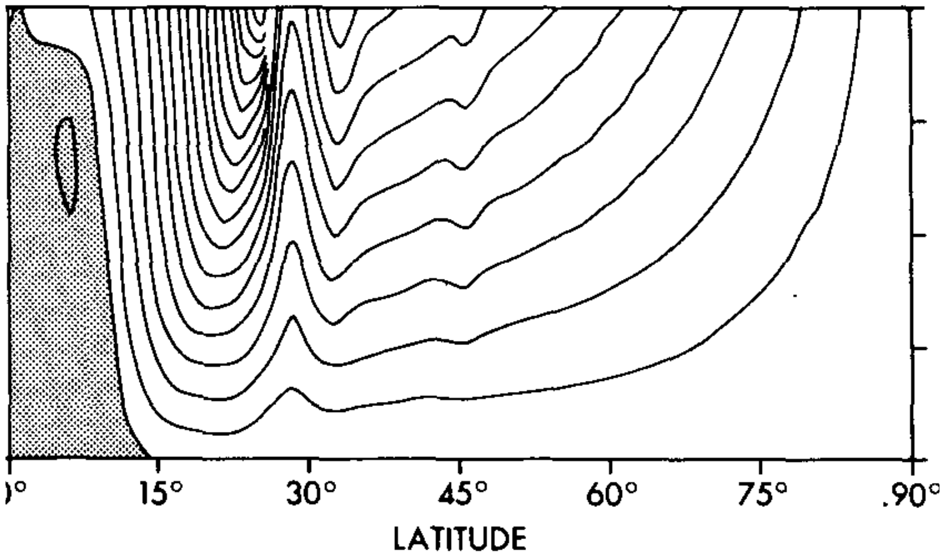


From Gierasch mechanism solution

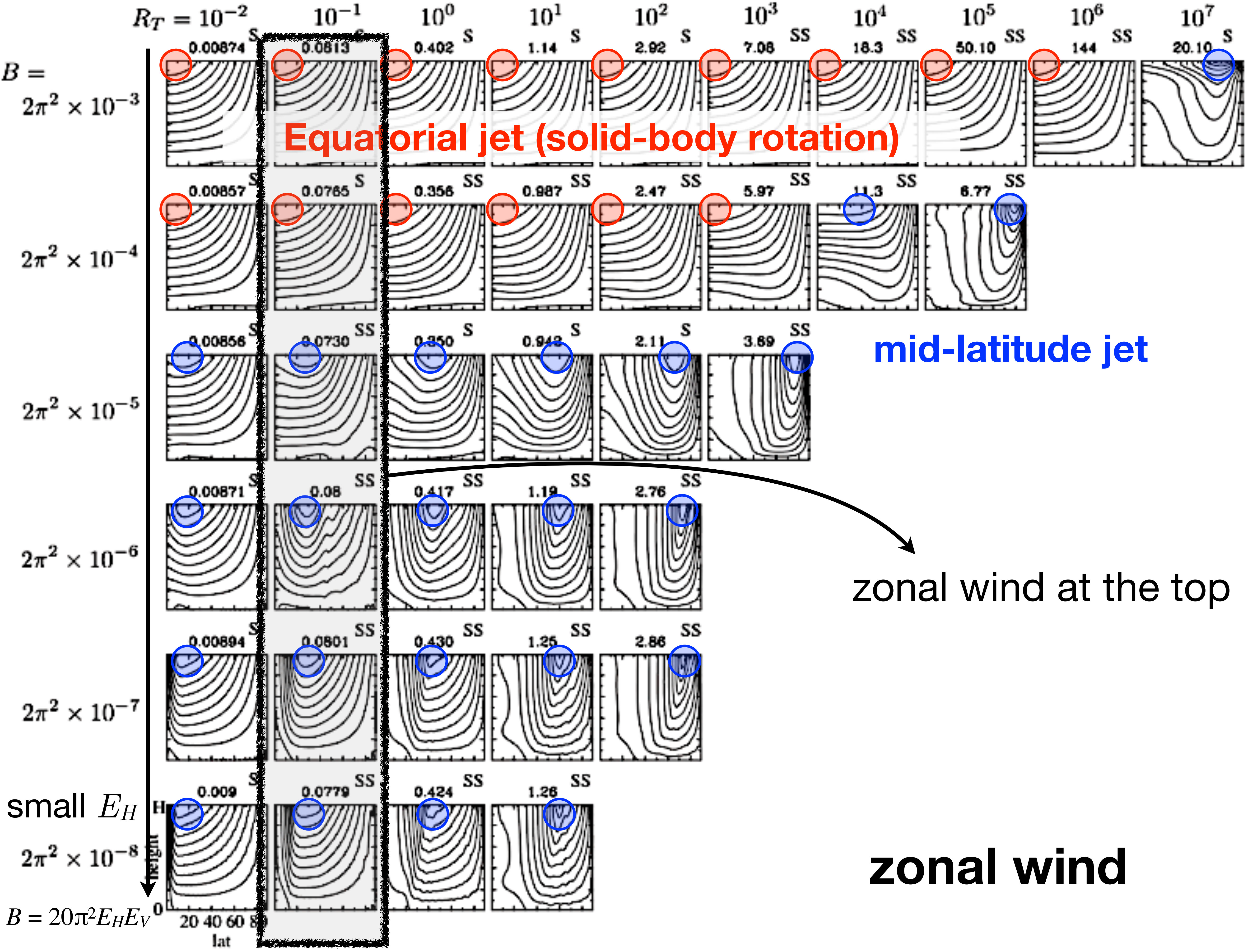
E_H : large

$E_H = 0$

to Held-Hou solution



Numerical experiments



Equatorial jet (solid-body rotation)

mid-latitude jet

zonal wind at the top

small E_H

$B = 20\pi^2 E_H E_V$

zonal wind

5 6 Zonal wind at the top

☐ **Equatorial jet sol.**

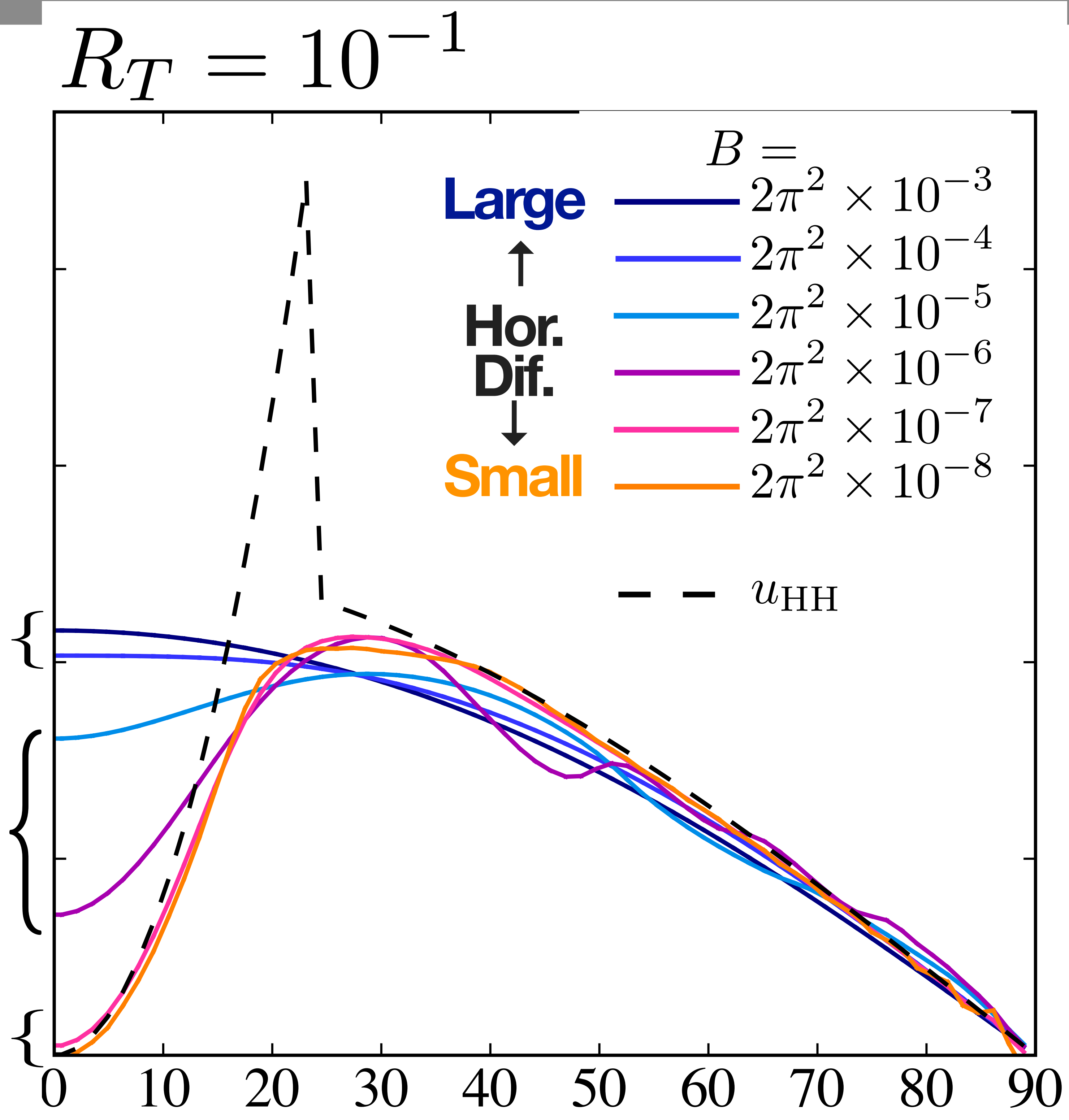
close to solid-body

▨ **Mid-latitude jet sol.**

$u \not\approx u_{\mathrm{HH}}$

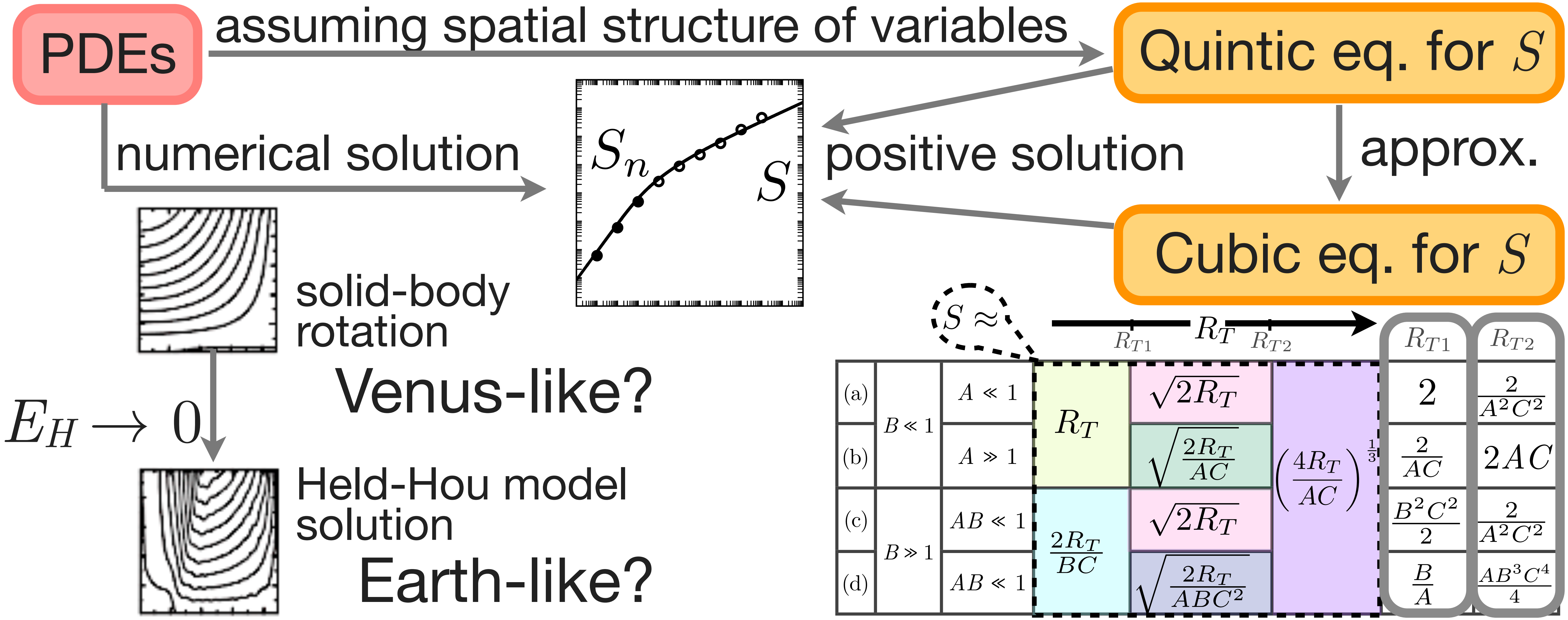
■ **Held-Hou model sol.**

$u \approx u_{\mathrm{HH}}$



⑥ Summary

- We explored the strength of the superrotation maintained by the Gierasch mechanism in an idealized axisymmetric boussinesq fluid model with strong horizontal diffusion.



PDEs

assuming spatial structure of variables

Quintic eq. for S

numerical solution

positive solution

approx.

Cubic eq. for S

solid-body rotation

Venus-like?

$E_H \rightarrow 0$

Held-Hou model solution

Earth-like?

			$S \approx$			R_{T1}	R_{T2}
(a)	$B \ll 1$	$A \ll 1$	R_T	$\sqrt{2R_T}$	$\left(\frac{4R_T}{AC}\right)^{\frac{1}{3}}$	2	$\frac{2}{A^2C^2}$
(b)		$A \gg 1$		$\sqrt{\frac{2R_T}{AC}}$		$\frac{2}{AC}$	$2AC$
(c)	$B \gg 1$	$AB \ll 1$	$\frac{2R_T}{BC}$	$\sqrt{2R_T}$		$\frac{B^2C^2}{2}$	$\frac{2}{A^2C^2}$
(d)		$AB \ll 1$		$\sqrt{\frac{2R_T}{ABC^2}}$		$\frac{B}{A}$	$\frac{AB^3C^4}{4}$

Ref.1 Yamamoto & Yoden (2013) Theoretical Estimation of the Superrotation Strength in an Idealized Quasi-Axisymmetric Model of Planetary Atmospheres, *J. Meteor. Soc. Japan*, 91(2) in print

Ref.2 Yamamoto et al. (2009) Axisymmetric Steady Solutions in an Idealized Model of Atmospheric General Circulations: Hadley Circulation and Super-rotation, Theor. Appl. Mech. Japan, 57, 147-158.