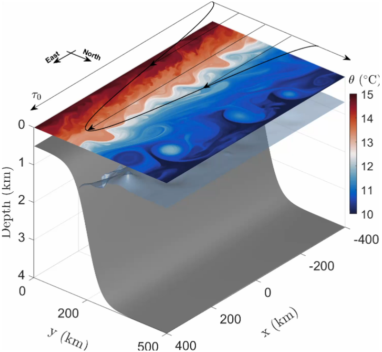
Baroclinic dynamics in the presence of slopes



Julian Mak (HKUST + NOC)

Overview

0. shelf seas and slopes, baroclinic eddies, and it's parameterisation

1. baroclinic turbulence over slopes

   → nonlinear simulations

   → parameterisations

   → GEOMETRIC: an eddy-mean interaction framework

   → parameterising suppression of fluxes

2. mechanism for slope suppression

   → linear instability point of view?

   → revisiting the (sloped) Eady problem

   → interpretation in terms of CRWs

   → GEOMETRIC analysis

Shelf seas + continental slopes

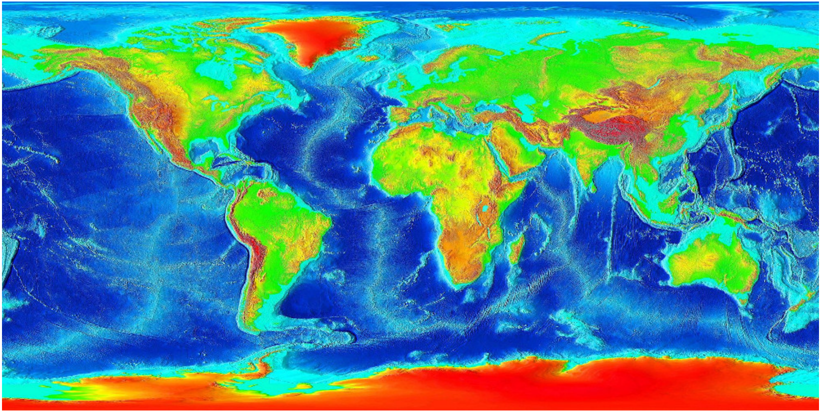
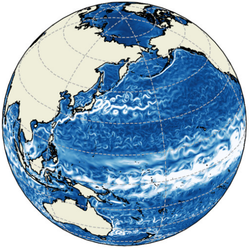
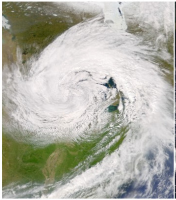


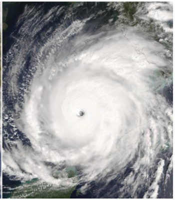
Figure: Locations of shelf seas denoted by the cyan colour. Taken from Wikipedia (`https://en.wikipedia.org/wiki/Continental_shelf`) made from NOAA data.

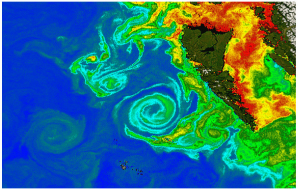
- exchange between shelves and open ocean important

Baroclinic eddies

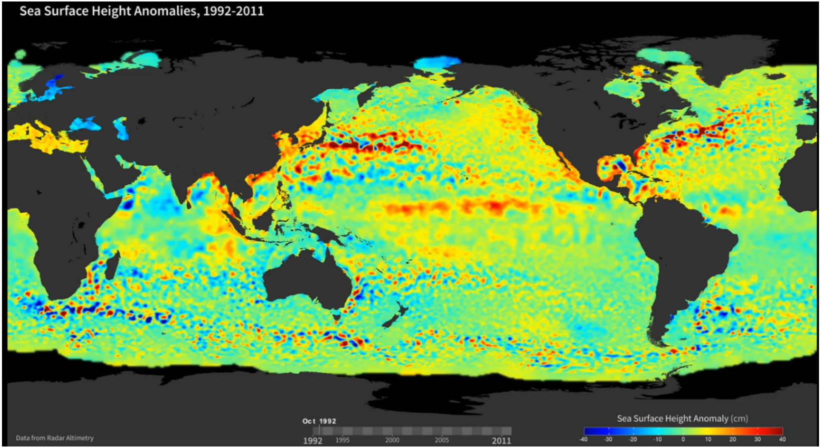




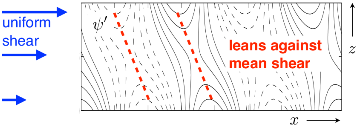




Baroclinic eddies



Baroclinic instability

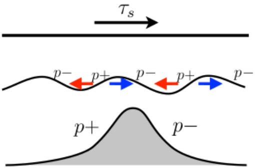


- baroclinic instability

  → reduces flow shear (⇒ reducing tilt in isopycnal / isentrope)

  → fueled by available potential energy

- also important for momentum transport



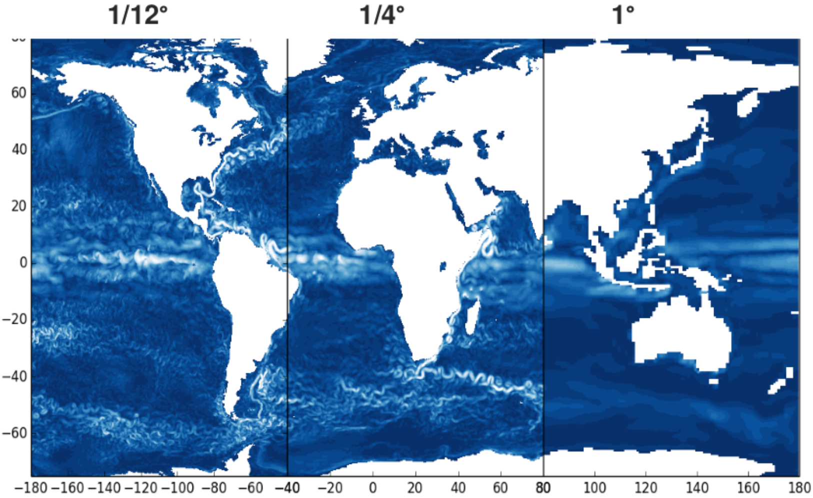
$$
\begin{aligned}
\overline{p'\nabla z'} &= -\overline{z'\nabla p'} \\
&= -\rho_0 f\,\overline{z'v'} \\
&= \frac{\rho_0 f}{N^2}\overline{b'v'}
\end{aligned}
$$

eddy form stress ⟷ eddy buoyancy flux

vertical momentum transfer ⟷ lateral heat transfer

(figures from David Marshall)

Parameterisation



from Helene Hewitt (UKMO)

Part 1: baroclinic turbulence over slopes

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RESEARCH ARTICLE
10.1029/2022MS003229

Scalings for Eddy Buoyancy Fluxes Across Prograde Shelf/ Slope Fronts

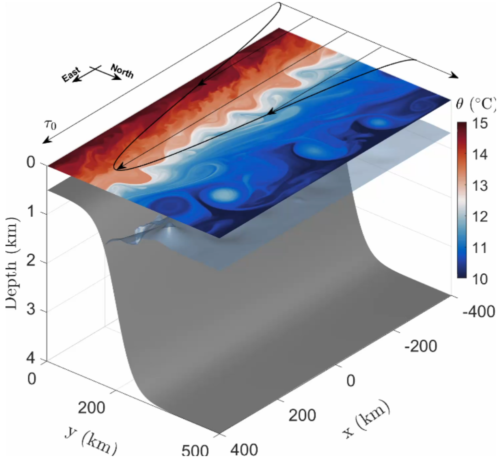
Huaiyu Wei[1,2], Yan Wang[1,2], Andrew L. Stewart[3], and Julian Mak[1,2]

Parameterizing Eddy Buoyancy Fluxes Across Prograde Shelf/Slope Fronts Using a Slope-Aware GEOMETRIC Closure

HUAIYU WEI,[a,b] YAN WANG,[a,b] AND JULIAN MAK[a,b,c]

- most of the following is Huaiyu's PhD work (currently post-doc at UCLA)

Baroclinic simulation over slopes



- wind forced simulation in MITgcm (2km resolution)

  → downwelling favourable wind forcing

  → strong jet along shelf break

  → eddies on and off shelf have different length-scales ($\sim L_d$) and different properties

Q. how to parameterise?

Parameterisation

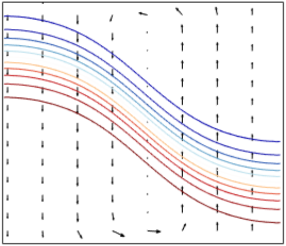
- **Gent–McWilliams** (GM) scheme:

$$\overline{\boldsymbol{u}'b'} = -\kappa_{\mathrm{gm}} \nabla \overline{b}$$

→ this one is really an **eddy-induced advection**

→ flattens isopycnals, parameterisation of **form stress**

(resembles but is not exactly thickness diffusion)



Gent & McWilliams (1990); Gent *et al.* (1995)

- widely used in ocean GCMs, many good things about it

  → positive-definite sink of APE

  → reduces spurious deep convection in models

Parameterisation

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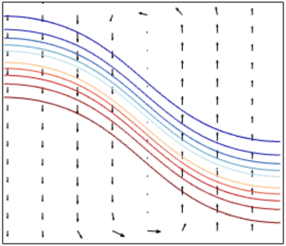


Gent & McWilliams (1990); Gent *et al.* (1995)

- widely used in ocean GCMs, many good things about it

  → positive-definite sink of APE

  → reduces spurious deep convection in models

- eddy energy E ↔ eddy activity, so

$$\kappa_{\mathrm{gm}} = \kappa_{\mathrm{gm}}(f(E), \ldots)\ ?$$

→ mixing length theory ⇒ $\kappa_{\mathrm{gm}} \sim \sqrt{E}$ (e.g. Eden & Greatbatch, 2008; Jansen *et al.*, 2015)

GEOMETRIC framework

GEOMETRIC framework

Under QG dynamics, mean equation may be written as

$$\frac{\partial \overline{\boldsymbol{u}}}{\partial t} + f(\overline{\boldsymbol{u}}) = \nabla \cdot \mathbf{E}, \qquad \mathbf{E} = \begin{pmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{pmatrix},$$

- rank 2 tensor $\mathbf{E}$ encodes all fluctuation quantities

$$M = \frac{1}{2}\overline{v'^2 - u'^2}, \quad N = \overline{u'v'},$$

$$P = \frac{1}{2N_0}\overline{b'^2},$$

$$R = \frac{f_0}{N_0^2}\overline{u'b'}, \quad S = \frac{f_0}{N_0^2}\overline{v'b'},$$

$\rightarrow$ Eliassen–Palm flux tensor

Q. parameterise in a symmetry-preserving way?

Marshall *et al.* (2012); Maddison & Marshall (2013) [see also Hoskins *et al.* (1983)]

Parameterisation: GEOMETRIC

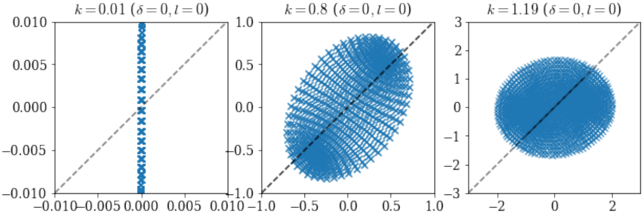


Figure: Demonstration of eddy variance ellipses for Eady problem (v' and b' here).

- consider geometric parameters relating to eddy variance ellipses

  $\rightarrow$ anisotropy parameters $\gamma_{b,m}$

  $\rightarrow$ angle parameters $\phi_{b,m}$

- note ϕ_m relates to actual eddy shape (cf. Tamarin *et al.*, 2016)

  $\rightarrow$ ϕ_b does not, but the vertical angle parameter $\tan 2\phi_t = \gamma_b \tan 2\lambda$ does (e.g. Youngs *et al.*, 2017)

Parameterisation: GEOMETRIC

$$M = \frac{1}{2}\overline{v'^2 - u'^2} = -\gamma_m E \cos 2\phi_m \cos^2\lambda, \quad N = \overline{u'v'} = \gamma_m E \sin 2\phi_m \cos^2\lambda,$$

$$P = \frac{1}{2N_0}\overline{b'^2} = E \sin^2\lambda,$$

$$R = \frac{f_0}{N_0^2}\overline{u'b'} = \gamma_b \frac{f_0}{N_0} E \cos\phi_b \sin 2\lambda, \quad S = \frac{f_0}{N_0^2}\overline{v'b'} = \gamma_b \frac{f_0}{N_0} E \sin\phi_b \sin 2\lambda,$$

with geometric parameters

$$\gamma_m = \frac{\sqrt{M^2 + N^2}}{K}, \qquad \gamma_b = \frac{N_0}{2f_0}\sqrt{\frac{R^2 + S^2}{KP}},$$

$$\sin 2\phi_m = \frac{N}{\sqrt{M^2 + N^2}}, \qquad \sin\phi_b = \frac{S}{\sqrt{R^2 + S^2}},$$

$$\frac{K}{E} = \cos^2\lambda, \quad \frac{P}{E} = \sin^2\lambda, \quad \tan^2\lambda = \frac{P}{K}.$$

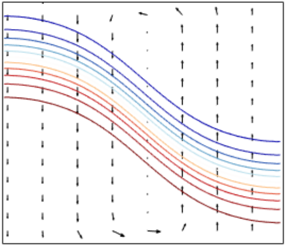
Marshall *et al.* (2012); Maddison & Marshall (2013)

GEOMETRIC framework

- GM scheme close for buoyancy fluxes

$$R = \frac{f_0}{N_0^2}\overline{u'b'} = \gamma_b \frac{f_0}{N_0} E \cos\phi_b \sin 2\lambda,$$

$$S = \frac{f_0}{N_0^2}\overline{v'b'} = \gamma_b \frac{f_0}{N_0} E \sin\phi_b \sin 2\lambda.$$



- $\|\mathbf{E}\|^2 \leq E$, tensor may be bounded in terms of eddy energy, and bound implies

$$\kappa_{\rm gm} = \alpha E \frac{(\partial \overline{b}/\partial z)^{1/2}}{|\nabla \overline{b}|^2} \equiv \alpha E \frac{N}{M^2},$$

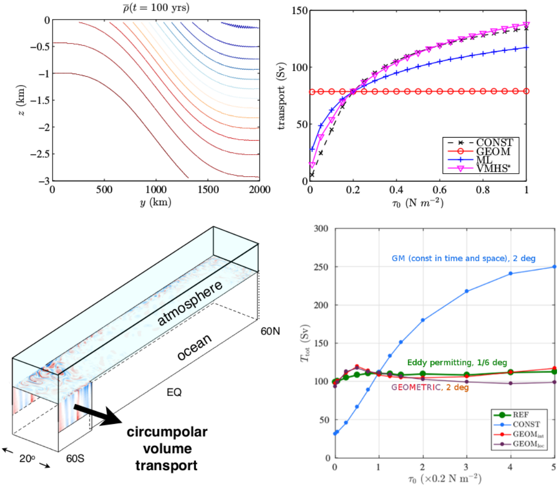
- $\alpha \sim \gamma_b \sin 2\lambda(\cos\phi_b, \sin\phi_b)$ is **non-dimensional** and $|\alpha| \leq 1$!

  $\rightarrow$ eddy efficiency parameter, **tunable** in parameterisations

  $\rightarrow$ closed by including a prognostic eddy energy budget for E

Marshall *et al.* (2012); Maddison & Marshall (2013); Mak *et al.* (2017, 2018, 2022a,b)

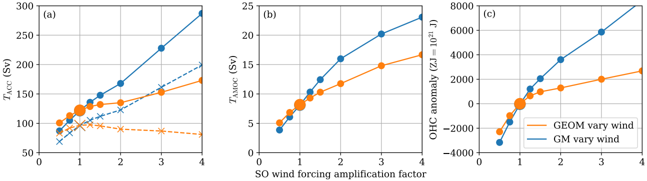
Parameterisation: GEOMETRIC

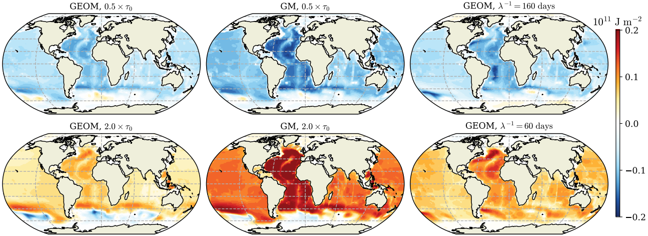


- get eddy saturation (mathematical reasons for this, ask me if interested)

Mak *et al.* (2017, 2018)

Parameterisation: GEOMETRIC



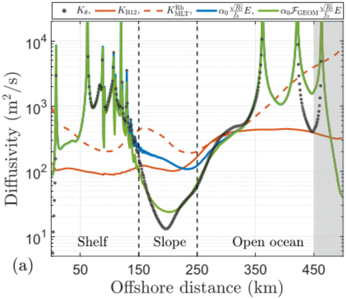


- reduces sensitivity of ocean heat content in ‘realistic’ model

(NEMO ORCA2 here, but ‘works’ also in ORCA1)

Mak *et al.* (2022a)

GEOMETRIC over slopes (Wei, *et al.*, 2022)

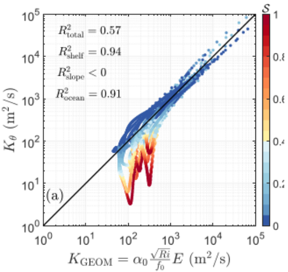


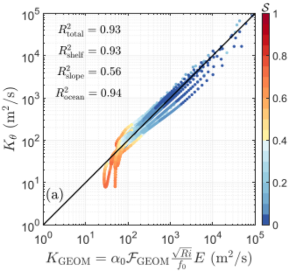
- diagnosed diffusivity 'suppressed' over slope region

  → least-squares type fitting for an over-determined system (Bachman & Fox-Kemper, 2013)

- suppression function fitted as

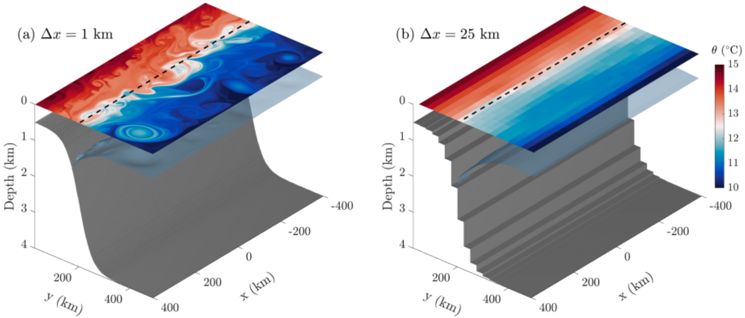
$$\mathcal{F}_{GEOM}(S) = \frac{1}{\mu_1 S^{\mu_2} + 1}.$$



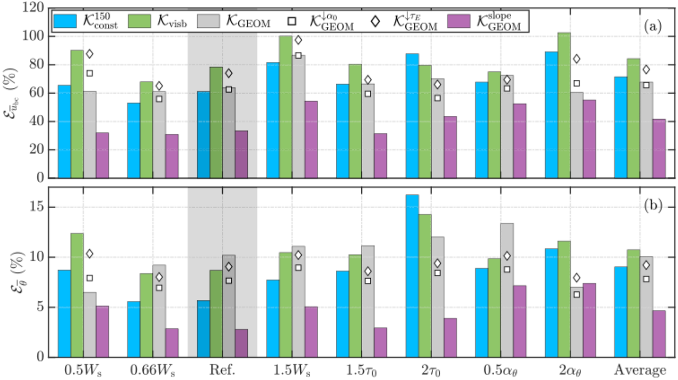


GEOMETRIC over slopes (Wei, *et al.*, 2024)

- suppressed GEOMETRIC ok from diagnostics, but in prognostic runs?



GEOMETRIC over slopes (Wei, *et al.*, 2024)



- relative errors of new variant is lowest

  → nonlinear feedbacks, κ_{gm} too large over shelves has effect over the domain via a 'pivot' mechanism

Summary

- introduced GEOMETRIC framework

  → parameterisation in terms of geometric parameters

  → a framework for analysing eddy-mean interactions

  → key role of the α eddy efficiency parameter

- suppression of eddy-mean interaction over slopes from simulations

  → can be represented through suppression of α (Wei *et al.*, 2022)

  → functions reasonably well in prognostic calculations (Wei *et al.*, 2024)

  → ongoing work to see impacts in global models

  → recent **experimental** evidence for suppression of α (Cheng *et al.*, 2025)

Part 2: mechanism for slope suppression

PHYSICAL REVIEW FLUIDS **9**, 083905 (2024)

Editors' Suggestion

Edge-wave phase shifts versus normal-mode phase tilts in an Eady problem with a sloping boundary

J. Mak* N. Harnik E. Heifetz G. Kumar† E. Q. Y. Ong

- focus on **linear instability** of (modified) Eady problem

Recap: suppression over slopes

- suggested suppression is

$$\kappa_{\text{gm}} = \alpha \mathcal{F}_{\text{GEOM}}(S) E \frac{N}{M^2}, \qquad \mathcal{F}_{\text{GEOM}}(S) = \frac{1}{\mu_1 S^{\mu_2} + 1},$$

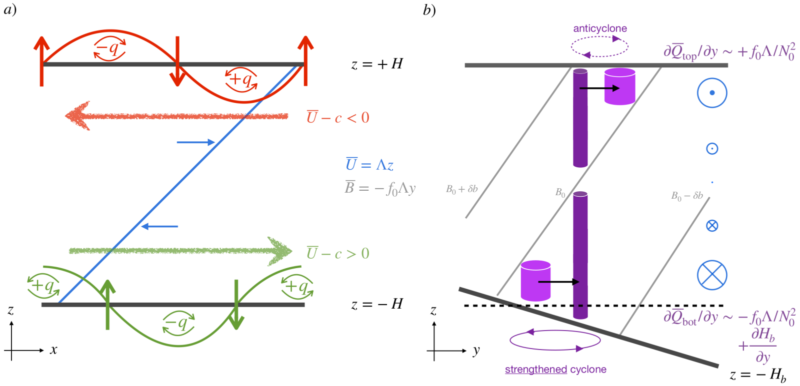
- from simulation results, it's not E or N/M^2 that are suppressed, so

$$\alpha \mapsto \alpha \mathcal{F}_{\text{GEOM}}(S) \text{ ?}$$

Q. which part of $\alpha \sim \gamma_b \sin 2\lambda (\cos \phi_b, \sin \phi_b)$ is being suppressed?

Q. why? mechanisms?

In the presence of a slope...



- consider **linear instability** point of view
  $\rightarrow$ modified Eady problem with a slope

figure inspired from Chen *et al.* (2020)

The equation

- standard QG, linear shear flow in vertical, $\boldsymbol{u} = U\boldsymbol{e}_x = \Lambda z/H$, be wise and linearise:

$$\left(\frac{\partial}{\partial t} + \Lambda z \frac{\partial}{\partial x}\right)\left(\nabla^2\psi + \frac{f_0^2}{N_0^2}\frac{\partial^2\psi}{\partial z^2}\right) = 0, \qquad z \in (-H, H),$$

$$\left(\frac{\partial}{\partial t} + \Lambda \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - \frac{\Lambda}{H}\frac{\partial\psi}{\partial x} = 0, \qquad z = H,$$

$$\left(\frac{\partial}{\partial t} - \Lambda \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - \left(\frac{\Lambda}{H} - \frac{N_0^2}{f_0}\frac{\partial H_b}{\partial y}\right)\frac{\partial\psi}{\partial x} = 0, \qquad z = -H.$$

The equations

- sensible non-dimensionalisation (!!)

$$\left(\frac{\partial}{\partial t} + z\frac{\partial}{\partial x}\right)\left(\nabla^2\psi + F^2\frac{\partial^2\psi}{\partial z^2}\right) = 0, \qquad z \in (-1, 1),$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - \frac{\partial\psi}{\partial x} = 0, \qquad z = 1,$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - (1-\delta)\frac{\partial\psi}{\partial x} = 0, \qquad z = -1,$$

- with $F^2 = (fL/NH)^2$, and key parameter is

$$\delta = \frac{\partial H_b}{\partial y} \Big/ \frac{\partial\rho/\partial y}{-\partial\rho/\partial z}$$

The equations

- modal solutions, interior PV equations imply

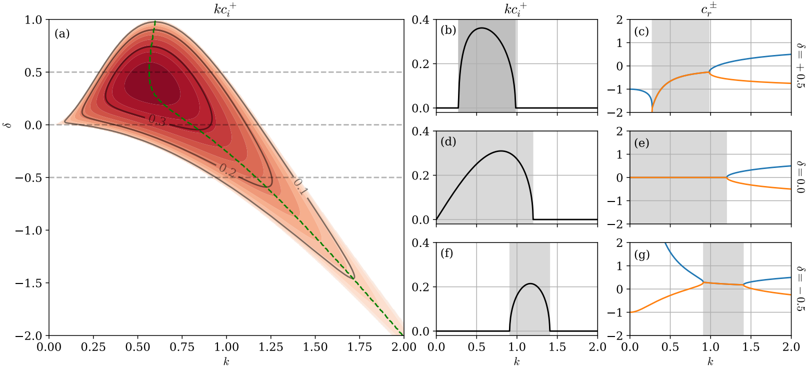
$$\tilde{\psi}(z) = a \cosh \mu z + b \sinh \mu z, \qquad \mu^2 = (k^2 + l^2)/F^2$$

- boundary conditions fix the constants a and b, leading to ($C = \cosh \mu$ and $S = \sinh \mu$)

$$0 = c^2 + \frac{\delta}{2\mu}\left(\frac{C}{S} + \frac{S}{C}\right) c + \frac{\delta^2}{4\mu^2} - \left(\frac{1 - \delta/2}{\mu} - \frac{C}{S}\right)\left(\frac{1 - \delta/2}{\mu} - \frac{S}{C}\right)$$

$\rightarrow$ solve analytically/numerically

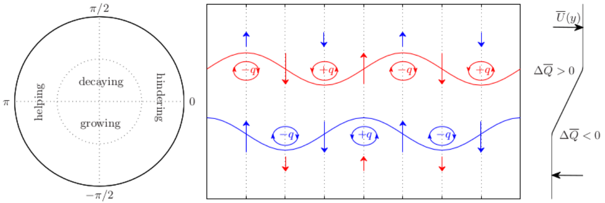
Instability characteristics



- reduced growth rates when $\delta < 0$ ('prograde' case)
- reduced bandwidth when $\delta > 0$, shuts off when $\delta \geq 1$

Q. mechanism?

CRW mechanism



- each CRW can interfere with each other

  → domain of influence ∼ Green's function

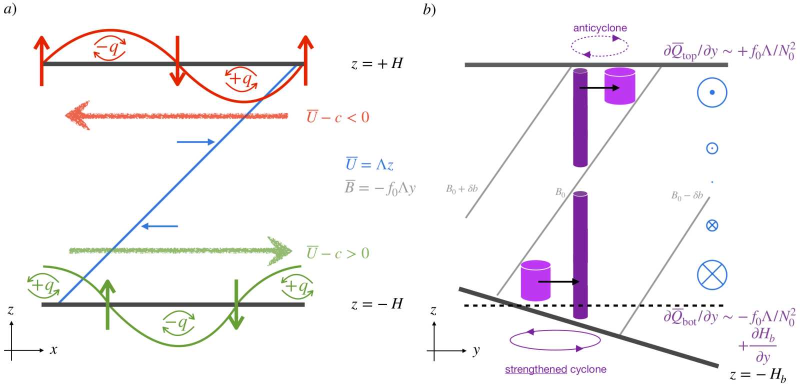
  → can affect amplitude and propagation

- phase-locking?

  → from mean flow and other wave

  → **modal instability** if phase-locked in constructively interfering configuration

In the presence of a slope...



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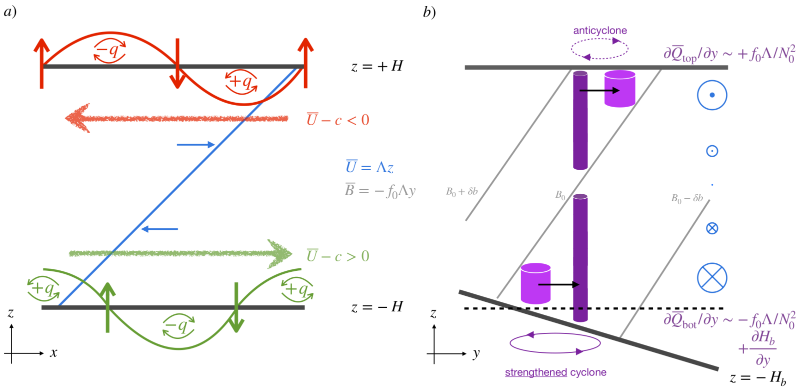


- argument in terms of CRWs: $\delta < 0$ makes bottom wave **faster**

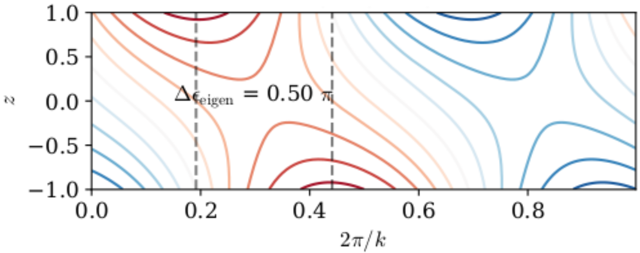
  $\Rightarrow$ bottom wave is such that $(U + c) \nearrow$

  $\Rightarrow$ for phase locking, want upper $(U - c) \nearrow$, so upper $c \searrow$

  $\Rightarrow$ Rossby waves $c \sim k^{-1}$, so want a larger k...? (Chen *et al.*, 2020)

In the presence of a slope...

- standard Eady problem (no slope), 2d problem ($l = 0$)
  $\rightarrow$ show normal-mode streamfunction $\psi = \tilde{\psi}(z)e^{i(kx-ct)}$
  $\rightarrow$ leans into shear, diagnosed $\Delta\epsilon_{\text{eigen}} = \pi/2$



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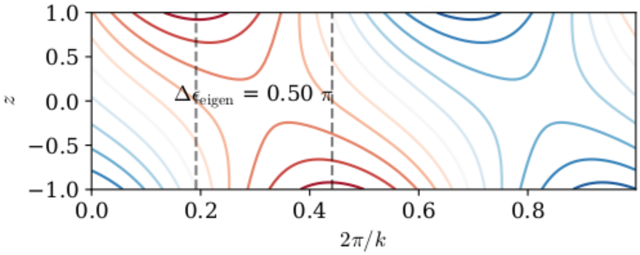

Q. does the **phase-tilt** change with δ?

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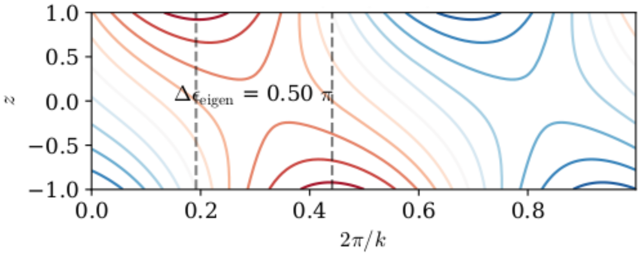


Q. does the **phase-tilt** change with δ?

Q. is the normal-mode phase-tilt even the right thing to look at, since we are talking about CRWs which are **edge-waves**?

Edge-wave basis

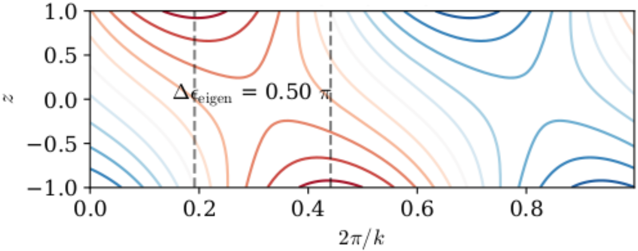


Figure: Streamfunction eigenfunction of most unstable mode in standard Eady problem.

- eigenfunction as a superposition of CRWs, $\psi = \psi_T + \psi_B$?

  $\rightarrow$ combination, would like it in edge-wave basis $\psi_{T,B}$ or $q_{T,B}$

Q. normal-mode **phase-tilt** is $\pi/2$, but is it really $\pi/2$ in the edge-waves **phase-shift**?

Edge-wave basis

- suppose $\tilde{\psi} = \tilde{\psi}_T + \tilde{\psi}_B$, and that $\tilde{q} = \tilde{q}_T + \tilde{q}_B$
- modal solutions:

$$\tilde{q} = -\mu^2 \tilde{\psi} + \frac{\partial^2 \tilde{\psi}}{\partial z^2},$$

with bcs (cf. Davies & Bishop, 1994, no buoyancy perturbation on other boundary)

$$\left.\frac{\partial \tilde{\psi}_T}{\partial z}\right|_{z=-1} = 0, \qquad \left.\frac{\partial \tilde{\psi}_B}{\partial z}\right|_{z=+1} = 0.$$

- suppose we demand localised PV signature from edge-waves with ($\hat{\delta}$ is the Dirac δ-distribution)

$$\tilde{q}_B = \hat{q}_B(t)\hat{\delta}(z+1), \qquad \tilde{q}_T = \hat{q}_T(t)\hat{\delta}(z-1),$$

then from the Green's function we have

$$\tilde{\psi}_B = -\hat{q}_B \frac{\cosh \mu(1-z)}{\mu \sinh 2\mu}, \qquad \tilde{\psi}_T = -\hat{q}_T \frac{\cosh \mu(1+z)}{\mu \sinh 2\mu}.$$

Edge-wave basis

- take previous thing, with $\hat{q}_T = T\mathrm{e}^{\mathrm{i}\epsilon_T}$ and $\hat{q}_B = B\mathrm{e}^{\mathrm{i}\epsilon_B}$, shove it into linearised EOM, tedious algebra gives

$$
\begin{aligned}
\frac{1}{T}\frac{\partial T}{\partial t} &= +\frac{k}{\mu\sinh 2\mu}\frac{B}{T}\sin\Delta\epsilon,\\
\frac{1}{B}\frac{\partial B}{\partial t} &= -\frac{k(1-\delta)}{\mu\sinh 2\mu}\frac{T}{B}\sin\Delta\epsilon,\\
-\frac{1}{k}\frac{\partial \epsilon_T}{\partial t} &= +\left[1-\frac{1}{\mu\sinh 2\mu}\left(\cosh 2\mu+\frac{B}{T}\cos\Delta\epsilon\right)\right],\\
-\frac{1}{k}\frac{\partial \epsilon_B}{\partial t} &= -\left[1-\frac{(1-\delta)}{\mu\sinh 2\mu}\left(\cosh 2\mu-\frac{T}{B}\cos\Delta\epsilon\right)\right],
\end{aligned}
$$

- $\Delta\epsilon = \epsilon_T - \epsilon_B$, edge-wave phase-shift

  $\rightarrow \Delta\epsilon > 0$ means the top *lags* bottom

Edge-wave basis

- more illuminating if written in terms of amplitude ratio $\tan\gamma = T/B$ and phase shift:

$$\frac{\partial\gamma}{\partial t} = \frac{k}{\mu\sinh 2\mu}\sin\Delta\epsilon(\cos 2\gamma + \delta\sin^2\gamma),$$

$$\frac{\partial\Delta\epsilon}{\partial t} = \frac{2k}{\mu\sinh 2\mu}\left[\left(1-\frac{\delta}{2}\right)\cosh 2\mu - \mu\sinh 2\mu + \left(\frac{1}{\sin 2\gamma} - \frac{\delta}{2}\tan\gamma\right)\cos\Delta\epsilon\right].$$

- two-dimensional dynamical system

  $\rightarrow$ analysis of phase portraits, related to transient/non-modal growth

  $\rightarrow$ synchronised growth/decay related to modal instabilities, or fixed points of the system

  $\rightarrow$ bifurcations (Hopf bifurcation here...?)

Edge-wave basis

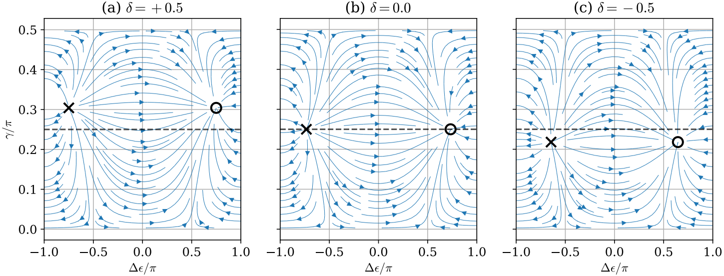


Figure: Phrase portrait for the case of $\delta = +0.5$, $\delta = 0$, and $\delta = -0.5$.

- $\delta < 0$ is where bottom wave is stronger ($B > T$)
  $\rightarrow \gamma = \arctan(T/B) < \pi/4$, i.e. $B < T$, consistent, and vice-versa
  $\rightarrow$ in fact, for synchronised growth, we should have

$$\left|\frac{1}{\tan\gamma}\right| = \left|\frac{B}{T}\right| = \sqrt{1-\delta},$$

- BUT $\Delta\epsilon \neq \pi/2$ (not remotely close!)

Edge-wave basis

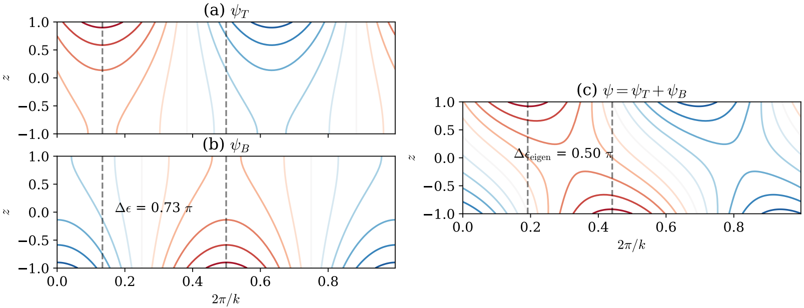


Figure: Edge-wave phase-shift vs normal-mode phase-tilt.

- mutual interaction matters!

  $\rightarrow$ in terms of mutual wave propagation, and constructive interference

  $\rightarrow$ $\pi/2$ is for optimal constructive interference, but not necessarily optimal for phase-locking (joint consideration required)

- $\Delta\epsilon_{\text{eigen}} = \pi/2$ is a phase-**tilt**

Edge-wave basis

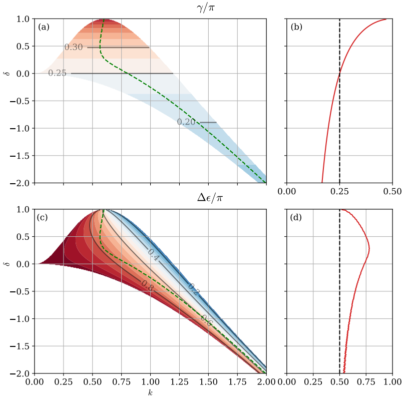


Figure: Amplitude ratio and edge-wave phase-shift over parameter space.

- amplitude ratio exactly as predicted varying with δ, and physically consistent ($\delta < 0$ has $B > T$)
- phase shifts expected for fixed δ varying k

  $\rightarrow$ in phase for k small, because interaction strong (and vice-versa)
- explanations just in terms of phase-locking incomplete

- strength of interaction $\Rightarrow$ phase shift and phase locking

  $\rightarrow \delta \searrow -\infty$, $B \nearrow$, interaction $\nearrow$, can offset by $k \nearrow$

Edge-wave basis

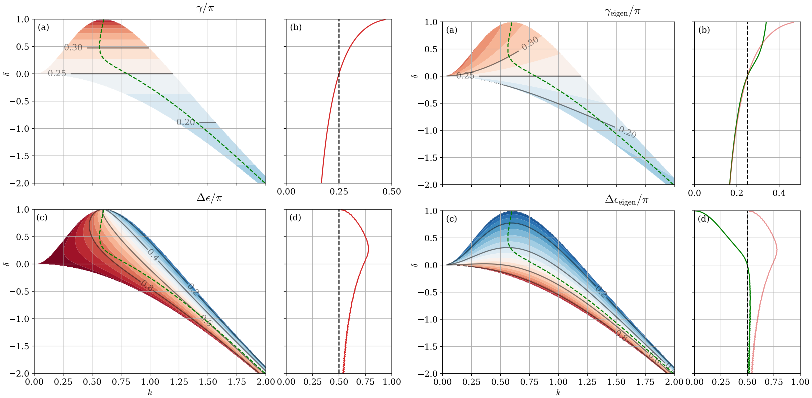


Figure: Amplitude ratio and edge-wave phase-shift over parameter space, from (left) edge-waves and (right) eigenfunction itself.

- $\Delta\epsilon_{\text{eigen}}$ as efficiency for APE extraction? (dubious)

Edge-wave basis

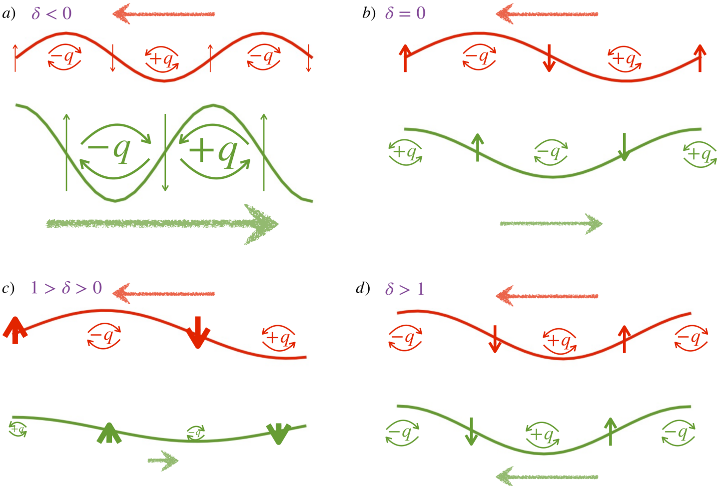


Figure: Schematic for δ and its effects on the edge-waves.

GEOMETRIC framework and links with CRWs?

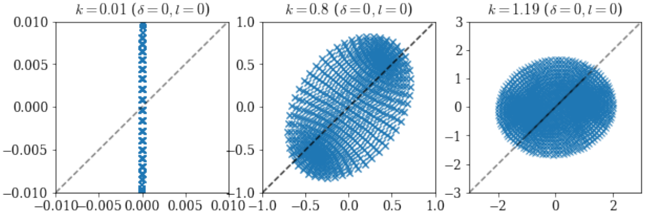


Figure: Demonstration of eddy variance ellipses for Eady problem (v' and b' here).

- for $l = 0$, i.e. no meridional variation, $u' = 0$, and so $R = N = 0$ while $M^2 = K$, and so

$$\gamma_m = 1, \quad \phi_m = 0, \quad \phi_b = \pm\frac{\pi}{2}, \quad \alpha = \pm\gamma_b \sin 2\lambda.$$

- $\phi_t \sim \Delta\epsilon$?

GEOMETRIC framework and links with CRWs

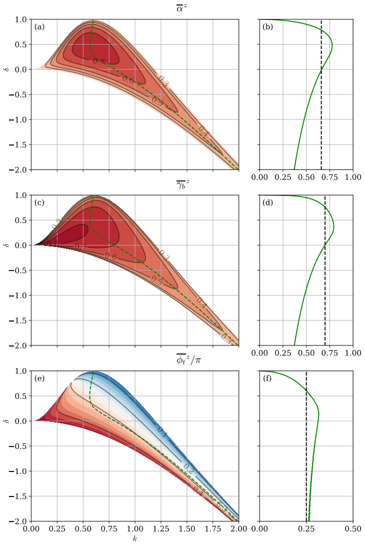


Figure: Geometric parameters for unstable modes.

- α very similar to the growth rate

  $\rightarrow$ proxy for eddy energy extraction?

  $\rightarrow$ α suppression mainly from eddy anisotropy γ_b (not shown; expected?)

- ϕ_t seems to have some relation with $\Delta\epsilon$ (cf. barotropic case of Tamarin *et al.*, 2016)

  $\rightarrow$ certainly better correlation than $\Delta\epsilon_{\text{eigen}}$

Summary

- cross slope suppression through inefficient instability mechanism?

  $\rightarrow$ really is α that is suppressed, suppresion through eddy anisotropy γ_b

Q. analytical links between GEOMETRIC and CRWs? (e.g. Tamarin *et* al., 2016)

  $\rightarrow$ suggestive here numerically, did not attempt derivation (laziness...)

- edge-wave basis equivalent and physically more informative

  $\rightarrow$ most works talk edge-waves phase-shifts, but present results for normal-mode phase-tilts

  $\rightarrow$ constructed system manually here, but can do this more generally (using orthogonality in e.g., pseudo-momentum; Held, 1985)

  $\rightarrow$ reduction to dynamical system formulation

Outlook (theory biased)

- Eady problem is $\mathcal{PT}$ symmetric, and several others are obviously (!?) $\mathcal{PT}$ symmetric

  $\rightarrow$ Kelvin–Helmholtz (Qin *et al.*, 2019)

  $\rightarrow$ (modified) Phillips problem (David *et al.*, 2022)

  $\rightarrow$ Eady with β, Rayleigh problem (HD and MHD version), ...

- links with CRWs?

  $\rightarrow$ phase-locking $\sim$ spontaneous $\mathcal{PT}$ symmetry breaking?

  $\rightarrow$ bifurcations and stability boundaries $\sim$ Krein collisions at exceptional points

? QM + QFT techniques applied to classical systems?

  $\rightarrow$ reality of spectrum (e.g. various works by Mostafazadeh) $\sim$ no phase-locking $\sim$ sufficient conditions for stability (e.g. Arnol'd 1966 etc.)?

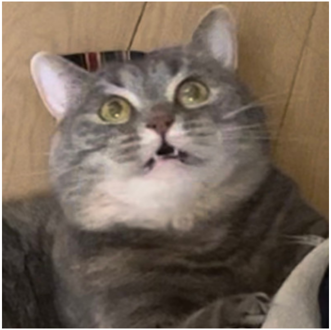
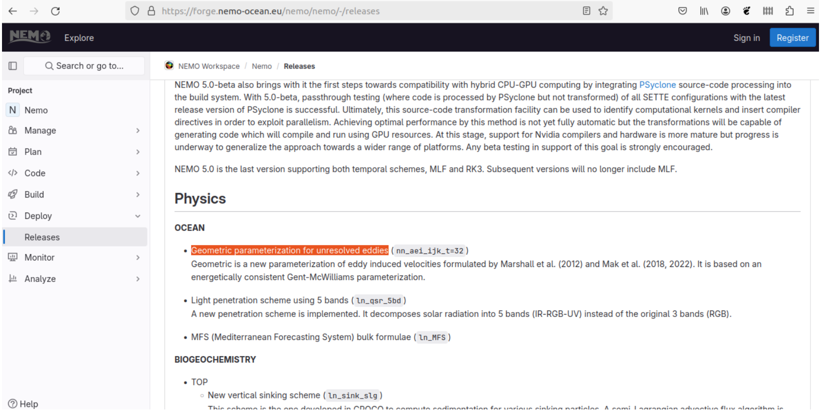


Figure: Questions?

Parameterisation: GEOMETRIC



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Help

NEMO 5.0-beta also brings with it the first steps towards compatibility with hybrid CPU-GPU computing by integrating PSyclone source-code processing into the build system. With 5.0-beta, passthrough testing (where code is processed by PSyclone but not transformed) of all SETTE configurations with the latest release version of PSyclone is successful. Ultimately, this source-code transformation facility can be used to identify computational kernels and insert compiler directives in order to exploit parallelism. Achieving optimal performance by this method is not yet fully automatic but the transformations will be capable of generating code which will compile and run using GPU resources. At this stage, support for Nvidia compilers and hardware is more mature but progress is underway to generalize the approach towards a wider range of platforms. Any beta testing in support of this goal is strongly encouraged.

NEMO 5.0 is the last version supporting both temporal schemes, MLF and RK3. Subsequent versions will no longer include MLF.

Physics

OCEAN

- Geometric parameterization for unresolved eddies (`nn_aei_ijk_t=32`)
  Geometric is a new parameterization of eddy induced velocities formulated by Marshall et al. (2012) and Mak et al. (2018, 2022). It is based on an energetically consistent Gent-McWilliams parameterization.
- Light penetration scheme using 5 bands (`ln_qsr_5bd`)
  A new penetration scheme is implemented. It decomposes solar radiation into 5 bands (IR-RGB-UV) instead of the original 3 bands (RGB).
- MFS (Mediterranean Forecasting System) bulk formulae (`ln_MFS`)

BIOGEOCHEMISTRY

- TOP
  - New vertical sinking scheme (`ln_sink_slg`)
    This scheme is the one developed in CROCO to compute sedimentation for various sinking particles. A semi-Lagrangian advective flux algorithm is

- in NEMO 5.0

  → also in MITgcm and MOM6

$\mathcal{PT}$ symmetry

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t} + z\frac{\partial}{\partial x}\right)\left(\nabla^2\psi + F^2\frac{\partial^2\psi}{\partial z^2}\right) = 0, && z \in (-1,1), \\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - \frac{\partial\psi}{\partial x} = 0, && z = 1, \\
&\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z} - (1-\delta)\frac{\partial\psi}{\partial x} = 0, && z = -1,
\end{aligned}
$$

- with $F^2 = (fL/NH)^2$, and key parameter is

Observation:

- parity symmetry $\mathcal{P}$, $(x,y) \mapsto (-x,-y)$, then $(\partial_x, \partial_y) \mapsto (-\partial_x, -\partial_y)$, velocity $(u,v) \mapsto (-u,-v)$, so streamfunction $\psi \sim \int u \, \mathrm{d}y \mapsto -(-\psi) = \psi$
- time reversal symmetry $\mathcal{T}$, $t \mapsto -t$, then $\partial_t \mapsto -\partial_t$, $\psi \mapsto -\psi$ by analogous argument
- system above is $\mathcal{PT}$ symmetric (even number of minus signs to every term under the $\mathcal{PT}$ mapping)

$\mathcal{PT}$ symmetry (contd.)

- concept of $\mathcal{PT}$ symmetry in quantum mechanics + QFT
  $\rightarrow$ discrete symmetries
- operator $\mathcal{H}$ is $\mathcal{PT}$ symmetric if

$$(\mathcal{PT})\mathcal{H}^*(\mathcal{PT})^{-1} = \mathcal{H}, \tag{1}$$

  (* denotes complex and not Hermitian conjugate)
- interest in QM: $\mathcal{PT}$ systems can have a real spectrum even if they are non-Hermitian (e.g., Bender & Boettcher, 1998)
- Eady problem can be described as $c\phi = M\phi$ where

$$M = \frac{-1}{SC}\begin{pmatrix} \dfrac{\delta}{2\mu}C^2 & \left(1-\dfrac{\delta}{2}\right)\dfrac{CS}{\mu} - C^2 \\ \left(1-\dfrac{\delta}{2}\right)\dfrac{CS}{\mu} - S^2 & \dfrac{\delta}{2\mu}S^2 \end{pmatrix}, \tag{2}$$

  and M is $\mathcal{PT}$ symmetric (M is real and $\mathcal{PT} = -I$; latter from David *et al.*, 2022, PoF)

$\mathcal{PT}$ symmetry (contd.)

- if $c\phi = M\phi$, then for $\Delta = \mathrm{Tr}(M)^2 - 4\mathrm{Det}(M)$,

$$c^2 - \mathrm{Tr}(M) + \mathrm{Det}(M) = 0, \qquad c = \frac{1}{2}\left(\mathrm{Tr}(M) \pm \sqrt{\Delta}\right) \tag{3}$$

- if $\Delta < 0$, $c_i \neq 0$ (i.e. instability)
  $\to c_r^+ = c_r^-$ since $\sqrt{\Delta}$ purely imaginary
- if $\Delta > 0$, $c_i = 0$ (i.e. neutral)
  $\to c_r^+ \neq c_r^-$ since $\sqrt{\Delta}$ purely real
  $\to$ collision at $\Delta = 0$, exceptional points
- see David, Delplace & Venaille (2022), PoF for more